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Three-Lens Lattices for Extending the Energy Range of Non-scaling FFAGs

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Abstract

In this paper it is found that a three-quadrupole focussing system can be morphed continuously through FFD, FDF and DFF variants and back again while maintaining stable optics and even keeping the two transverse tunes constant. This relates to non-scaling FFAGs, where the magnet gradients define both the focussing and the variation of the field with momentum as the closed orbit sweeps across it. A two-lens focussing system cannot change the sign of either gradient without becoming unstable, meaning non-scaling FFAGs built with such a lattice eventually encounter too large a magnetic field at low energies. However, a theoretical system of magnet field variations using three lenses, with a potentially unlimited energy range and fixed tunes is presented here.

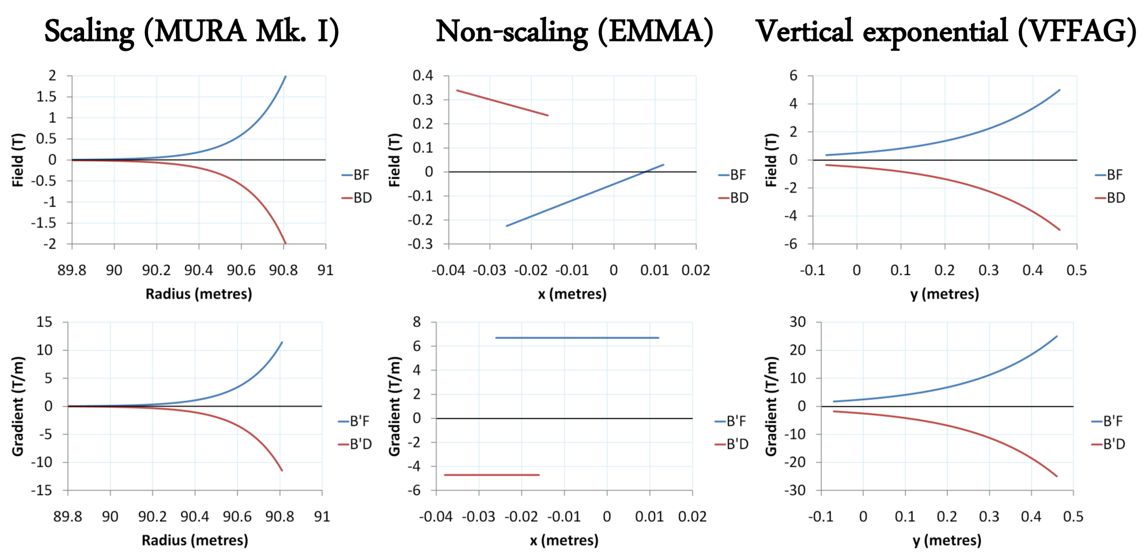
Table 1: Classification of FFAGs and their characteristics. Uppercase 'Y' indicates property is always true, lowercase 'y' that it is achievable in some cases. '3+' means three or more lenses per cell are required.

Type of FFAG	Fixed tunes	Wide *E* range	Isochronous	Small ring
Scaling	Y	Y	N	N
Non-scaling	3+	?	y	$y^\dagger$
Linear n.s.	N	N	y(quasi)	y
Vertical s.	Y	Y	N	N
V. n.s.	3+	?	?	?
Linear v.n.s.‡	?	?	?	?
Skew	y	?	$y^\dagger$	?

†Two 'y's may not be achievable simultaneously.

‡Linear field VFFAG suggested by D.J. Kelliher.

Field Profiles of existing two-magnet FFAGs



The Three-Dimensional "Necktie" Diagram

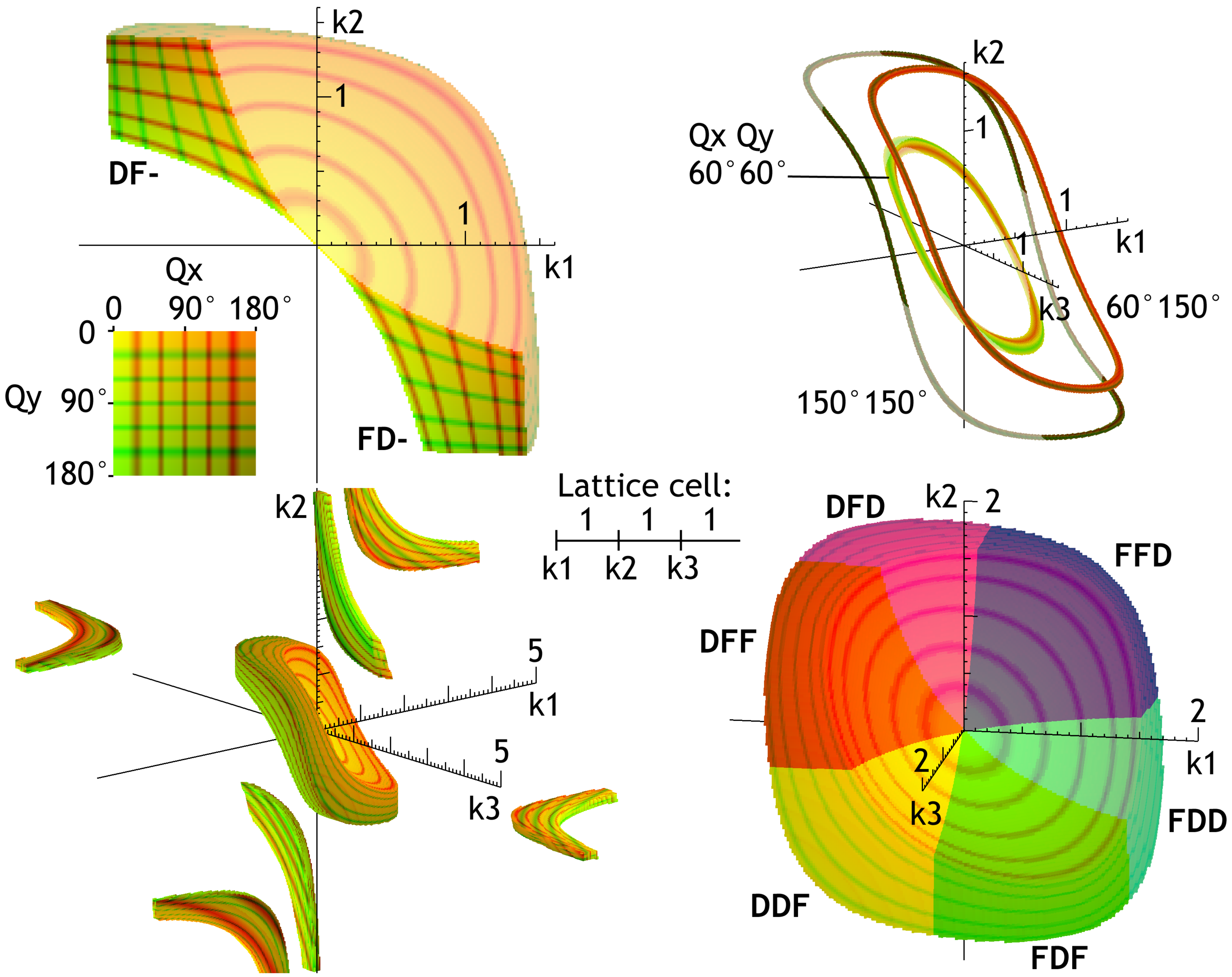
This is the three-dimensional 'stable volume' in (k_1,k_2,k_3)-space for a lattice with three quadrupole lenses of strength k_i separated by unit drifts. For any transverse cell tunes Q_x,Q_y there is a ring of possible lattices at that tune point. Quadrupoles may change from F to D without encountering unstable lattices (as happens with cells containing only two magnet types).

Top left: cut-away of the stable volume in the k_3=0 plane reveals two-dimensional necktie diagrams, coloured by cell tunes $Q_{x,y}$.

Bottom left: zooming out reveals secondary stable regions with tunes above π.

Top right: loci of constant tunes within the volume.

Bottom right: colouring by signs of the three lenses reveals a cycle of six lattice types.

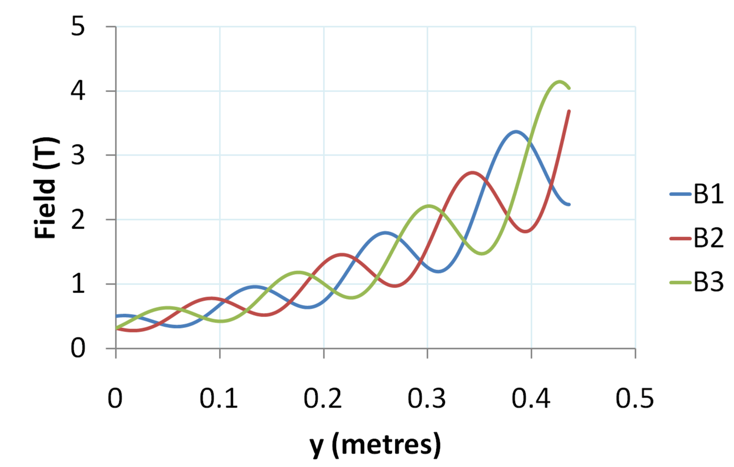


Oscillating Exponential Magnets

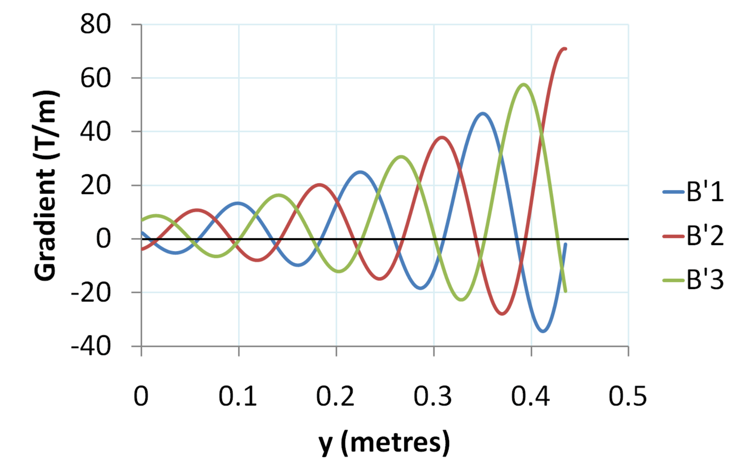
Combining the exponential field of the VFFAG with an oscillation to move the focussing system around the stable volume of the 3D necktie diagram suggests field profiles

$$B_{y,n}(0,y) = B_0 e^{ky}(1 + a\cos(wy + \varphi_n))$$

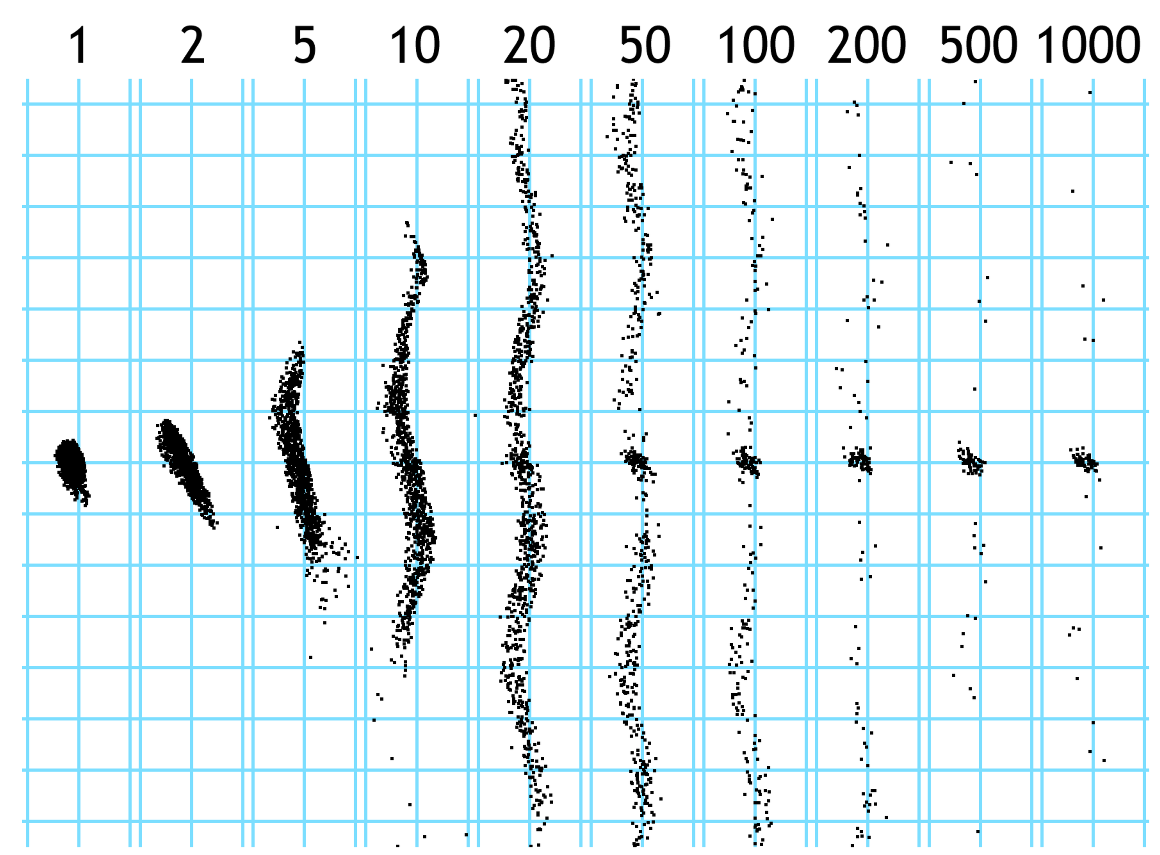
where a and w control the amplitude and frequency of the oscillations and φ_n is a lattice pseudo-phase that varies between magnets, so they do not all focus simultaneously. The e^{ky} term here could be replaced by the r^k law of horizontal scaling FFAGs to construct a variant with horizontal orbit excursion, where the areas of positive field 'spiral' outwards from one magnet to the next.



Left: field and gradient profile for a set of three oscillating exponential magnets with k=5m^{-1}, w=50m^{-1}, B_0=0.375T and a=1/3. The pseudo-phase φ=2π (n-1)/3 for the nth magnet. At least one of the gradients is negative at any position.



Below: weak vertical focussing behaviour in a lattice with three k=0, w=12m^{-1} oscillating exponential magnets. Grid is 10cm in (x,y), numbers are cells tracked. Particles are focussed horizontally by all lattices over several pseudo-phase cycles.



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Poster reference: WEPS086