VERTICAL ORBIT EXCURSION FFAG ACCELERATORS WITH EDGE FOCUSSING

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Abstract

FFAGs with vertical orbit excursion (VFFAGs) provide a promising alternative design for the magnets in fixed-field machines. They have a vertical magnetic field component that increases with height in the vertical aperture, yielding a skew quadrupole focussing structure. The end fields of such magnets with edge angles provide an alternating gradient without the need for reverse bends, thus reducing the machine circumference. Similarly to spiral scaling horizontal FFAGs (but unlike non-scaling versions), the machine has fixed tunes and no intrinsic limitation on momentum range. Rings capable of boosting the 800 MeV beam from the ISIS proton synchrotron (ϵ_{geom}=150 mm.mrad) to 3, 5 and 12 GeV using superconducting magnets are presented, the latter corresponding to 2.5 MW beam power.

MAGNET FIELD MODEL

The field within the body of a VFFAG magnet is given by $B_y = B_0 \mathrm{e}^{ky}$ on the $x = 0$ mid-plane. The beam travels in the $+z$ direction through the magnet and shifts to height $y = \frac{1}{k} \ln p/p_{\mathrm{inj}}$ as momentum p increases, so the injection orbit is at $y = 0$ and the current windings lie on the $\pm x$ sides of the vertical gap. Optics of a ring with such magnets without edge effects are considered in [1]. At injection, the magnet body has bending field B_0 and skew gradient $B_0 k$ (as well as higher multipoles of strength proportional to $B_0 k^{n\geq 2}$), so without edge effects B_0 must alternate in sign to provide alternating gradient focussing. k must be constant for the entire ring to satisfy the scaling law

$$y \mapsto y + \Delta y, \qquad (p, \mathbf{B}) \mapsto (p, \mathbf{B}) \mathrm{e}^{k\Delta y},$$

which ensures the orbit shape and tunes are preserved during acceleration. Having negative B_0 for some magnets produces reverse bends and increases machine circumference for a given field by ~5 times, similar to the circumference factor [2] in horizontal scaling FFAGs.

To represent magnets with edges, the parameter $\tau = \tan\theta_{\mathrm{edge}}$ is introduced, along with a coordinate $\zeta = z - \tau y$ so that the magnet corresponds to the region $0 \leq \zeta \leq L_{\mathrm{mag}}$ for all y. Field fall-off is determined by a function $f(\zeta)$ that approaches 1 in the magnet body and 0 outside. Naively one wants a mid-plane field $B_y = B_0 \mathrm{e}^{ky} f(\zeta)$ but to obey Maxwell's equation $(\nabla \times \mathbf{B})_x = 0$, this has to be modified to $(B_y, B_z) = B_0 \mathrm{e}^{ky} \left(f(\zeta) - \frac{\tau}{k} f'(\zeta), \frac{1}{k} f'(\zeta) \right)$. The note [3] derives this formula and the Taylor series extrapolation used to calculate fields for $x \neq 0$. For edge angles, $z \mapsto z + \tau \Delta y$ is added to the VFFAG scaling law to keep ζ constant (more accurately, this is a rotation of $\tau \Delta y / R$ about the ring centre).

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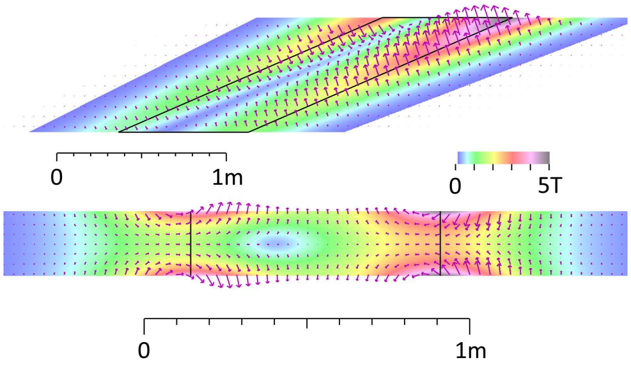

Figure 1: Cross-section of the 5 GeV ring magnet's field in ZY (top) and ZX (bottom) planes.

The resulting field is plotted in figure 1. The fringe field at the entrance to the magnet has opposite sign to that at the exit, providing alternating gradient focussing without changing the sign of B_0. Note that symmetry about the YZ plane forbids conventional quadrupole fields, meaning all focussing is skew apart from the solenoidal component B_z.

Field Enhancement Factor

As can be seen in figure 1, the largest fields are present in the magnet edges and off-plane. The field enhancement factor $\max_z |\mathbf{B}(x, y, z)|/(B_0 \mathrm{e}^{ky})$ is plotted in figure 2 (at $y = 0$, though by the scaling law it is the same at all y).

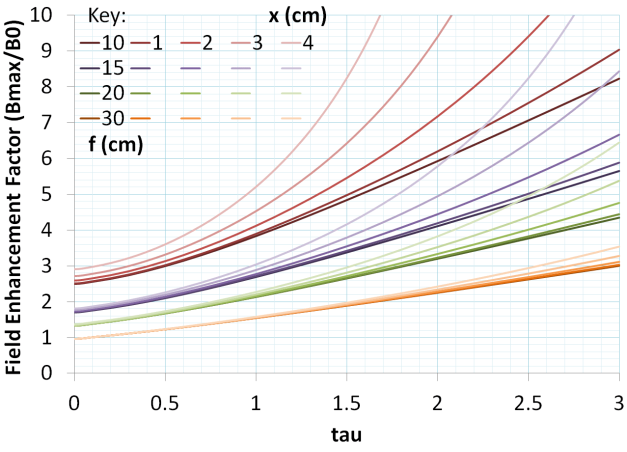

Figure 2: Field enhancements as a function of τ, fringe length (f) and distance from mid-plane (x) from 0 to 4 cm, in the 3 or 5 GeV magnet design with $k = 2.05\,\mathrm{m}^{-1}$.

Enhancement increases with τ but is ameliorated by increasing fringe length; it also increases extremely rapidly with x for small fringe lengths. However, it decreases with increasing k because higher k magnets actually have weaker fringe fields.

In ring design, this number fills a similar role to the circumference factor of scaling FFAGs: it is the amount a theoretical plain bending field must be multiplied by to find the real maximum field strength in a ring of fixed size and magnet filling factor.

PROTON ACCELERATOR RINGS

Parameters were sought for fixed-field rings to boost the energy of the two ISIS [4] proton bunches from 800 MeV, following the outline RF approach in [5]. Lattice cells containing a single VFFAG magnet and a reasonably-long drift space, with enough dynamic aperture for the 150 mm.mrad geometric emittance proton beam are given in table 1.

Table 1: VFFAG Proton Accelerator Ring Parameters

$E_{k,\mathrm{inj}}$	800 MeV		
$E_{k,\mathrm{ext}}$	3 GeV	5 GeV	12 GeV
Mean radius	52 m (2×ISIS)		
RF harmonic	$h = 8$		
Superperiods	80 (superperiod is one cell)		
Cell length	4.0841 m		
Drift length	3.3174 m		3.1257 m
Magnet Parameters			
Magnet length	0.7667 m		0.9584 m
B_0	0.5 T		0.4 T
k	$2.05\,\mathrm{m}^{-1}$		$2.23\,\mathrm{m}^{-1}$
$\tau = \tan\theta_{\mathrm{edge}}$	2.3		2.6
θ_{edge}	66.50°		68.96°
Fringe length	$f = 0.3\,\mathrm{m}$ in $B \propto \frac{1}{2} + \frac{1}{2}\tanh(z/f)$		
B_{ext}	1.3069 T	2.0036 T	3.5274 T
$B_{\mathrm{fringe}}/B_{\mathrm{body}}$	$2.7251_{x=4\,\mathrm{cm}}$		$2.6399_{x=2\,\mathrm{cm}}$
$B_{\max}$	3.5615 T	5.4600 T	9.3119 T
Beam Optics			
$y_{\mathrm{ext}} - y_{\mathrm{inj}}$	0.4687 m	0.6771 m	0.9762 m
μ_u (per cell)	71.11°		71.33°
μ_v	28.68°		19.65°
Q_u (ring)	15.802		15.851
Q_v	6.373		4.367

The beam power will increase in proportion to energy, so options are provided for neutron production at 3 GeV, high-power exotics production at 12 GeV and a 'compromise' energy of 5 GeV, which provides more power for neutrons but perhaps less efficiency. With the mean current 208 μA presently achievable in ISIS, these would have beam powers of 0.6, 2.5 and 1.0 MW respectively at 50 Hz.

The 12 GeV ring, the most aggressive design, with applications to neutrino factories and muon colliders, needed a slightly longer magnet to lower the peak field, which in turn required larger edge angles. The field enhancement has also been evaluated at $x = 2$ cm and not 4 cm to account for adiabatic shrinkage of the beam once accelerated to 12 GeV.

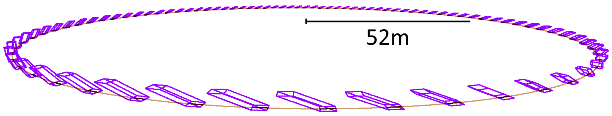

Figure 3: Perspective view of the 12 GeV ring.

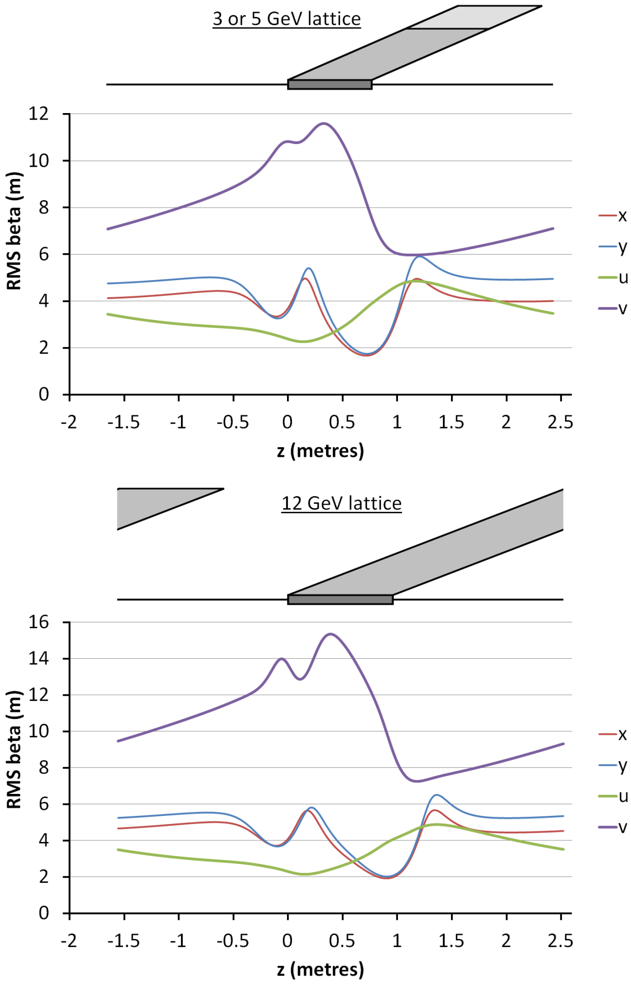

Figure 4: Beta functions in the two lattices, in non-skew and skew coordinates. Magnet size is to scale in z and y.

In terms of skew coordinates

$$u = (x + y)/\sqrt{2} \qquad \text{and} \qquad v = (y - x)/\sqrt{2},$$

the lattice beta functions shown in figure 4 are overall doublet-like but with some features in the end fields. The cell and machine tunes in table 1 are also given in terms of u and v. The x and y optics are highly coupled so do not behave like normal beta functions. Figure 5 shows how the phase spaces vary through the magnet, with some distortion of the matched shape, particularly in the (v, v') plane due to nonlinearity in the magnetic field.

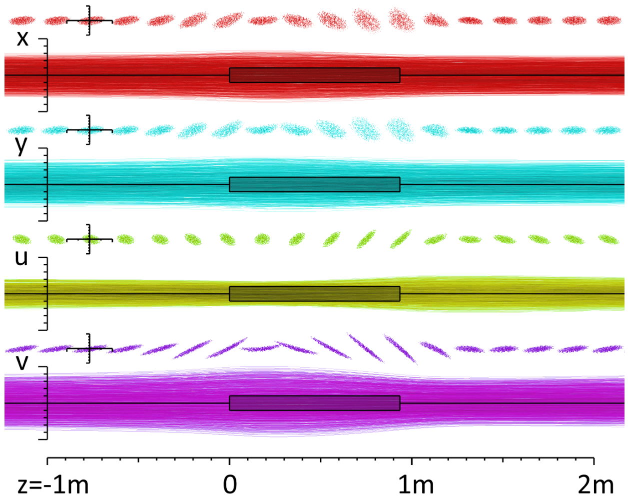

Figure 5: Phase space and beam evolution through the 12 GeV ring cell at injection energy. Transverse scale is ±5 cm and x', y', u', v' ranges are ±20 mrad.

The scaling law gives VFFAGs interesting properties, such as constant dispersions $D_x = 0$ and $D_y = \frac{1}{k}$ and a constant orbit length that makes $\gamma_{tr} = \infty$.

Dynamic Aperture Parameter Scan

The ring designs were found as combinations of six parameters $(B_0, k, \tau, f, L_{\text{mag}}, L_{\text{drift}})$, the last two being dictated by the integer RF harmonic number (ring circumference) and superperiodicity (cell length) together with B_0, which gives the magnet fill factor. The main focussing parameters k and τ were scanned over, producing plots like figure 6. For each square, 250 protons from a 150 mm.mrad waterbag beam were tracked for 250 cells and removed if $r > 10$ cm. Squares are coloured according to the percentage that survive, showing areas of good dynamic aperture.

High Intensity Issues

Since the ring tune is 80 times the cell tune, a fine-tuning stage is needed to steer the fractional parts of the ring tunes away from resonances. For the 3 or 5 GeV ring,

$$\frac{\partial Q_{u,v}}{\partial k} = \begin{bmatrix} -8.49 \\ -94.46 \end{bmatrix} \quad \text{and} \quad \frac{\partial Q_{u,v}}{\partial \tau} = \begin{bmatrix} 39.92 \\ 119.82 \end{bmatrix},$$

which are linearly independent enough to find any desired fractional ring tunes without major deterioration of the optics. This fine-tuning will also have to be done on the real machine, using trim coils producing fields proportional to $\partial\mathbf{B}/\partial k$ and $\partial\mathbf{B}/\partial\tau$.

The rapid variation of Q_v arises because the cell tune in v is quite close to zero. This is problematic since Q_v also varies rapidly in response to space charge forces, making the tune depressions of these rings roughly $\Delta Q_{sc,u} = -0.2$ and $\Delta Q_{sc,v} = -0.4$ at injection. This could be improved by finding rings with more balanced tunes or larger mean beta functions, though maybe at the expense of shorter drift spaces or a larger circumference.

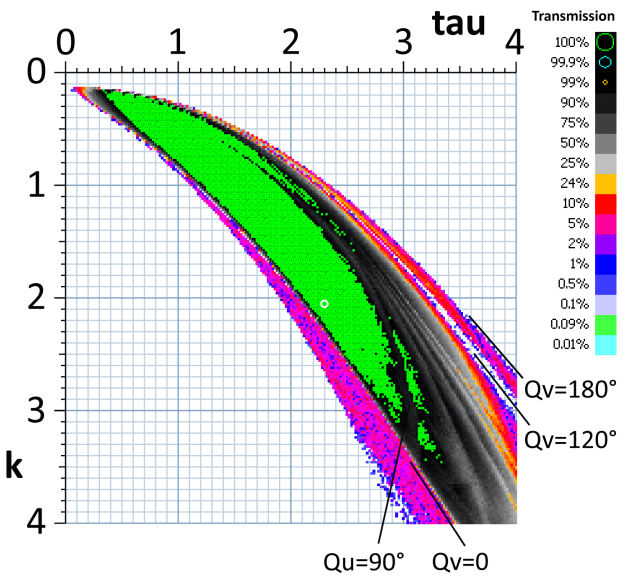

Figure 6: Proton beam transmission as a function of τ and k, with the 3 or 5 GeV ring design circled. Lines of increased loss correspond to cell tune resonances (labelled).

FUTURE WORK

- All tracking so far has been without space charge; these VFFAG rings should be further optimised for good behaviour at high current.
- Injection and (particularly) extraction needs to be studied in detail. A single drift length may not be long enough to produce the full deflection, so a distributed kicker system that works with the magnetic field near the top of the magnet may be required.
- Tracking and the magnetic field model both currently use a flat central reference surface ($x = 0$) whereas in reality this will be curved.
- Superconducting windings will have to complete the circuit at the top of the magnet: this can be done either by looping back on the same side, or across to the opposite side of the magnet where the current flows in the opposite direction. A full magnet geometry design will reveal which of these is best.

REFERENCES

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