Fixed-Tune Non-Scaling FFAs

Design for RAL FETS-FFA (3-12MeV p)

Analysis for cell of two thin lenses

Introduction

- Recent studies by Dejan Trbojevic have confirmed that non-scaling FFAs can have their tune dependence on momentum flattened by adding non-linear components to the magnet fields, although not necessarily for an unlimited momentum range.
- Quadrupole affects tune level, sextupole affects dQ/dp, octupole affects d^2Q/dp^2, etc.

FETS-FFA requirements

- 3-12MeV protons
- 4m average radius
- Several 1m drifts (one per cell here)
- Fixed tunes: $\Delta Q_{x,y}$<0.01 in ring, <0.001 per cell
- $Q_{x,y}$ tunable over full range of 1 in ring
  - For exploration of the tune plane in R&D
  - I chose 12 cells so this is 1/12 range per cell
- Geometrical aperture 1250mm.mrad (large)

Algorithm

- Runge-Kutta 4th order tracking step
- Loop to get trajectory in cell
- Finite difference to get transfer matrix
  - Also gives tunes if orbit is closed
- Iterate (Newton) to find closed orbit
- Loop over all FFA energies
- Finite difference parameters to get response matrix of tune functions to multipole changes
- Iterate (optimiser) to find fixed-tune lattice

Muon1

Muon1 cell optics mode

Muon1 FFA optics mode

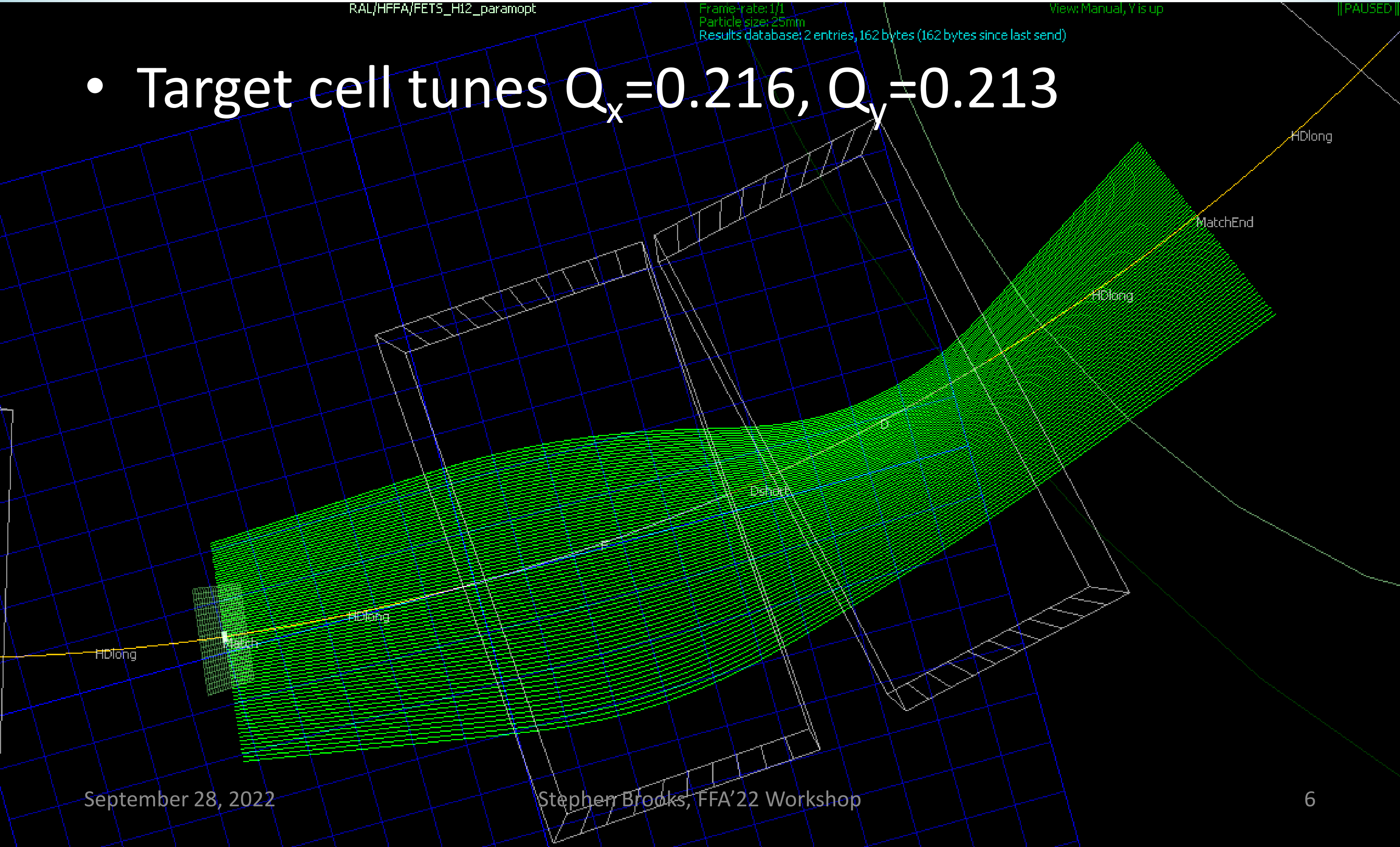
Fixed-tune FFA optimiser (parallel)

Field Model

- Midplane field is product of:
  - Transverse polynomial up to dodecapole
    - Rough initial field coefficients up to octupole were found by Muon1 genetic algorithm before using fixed-tune FFA optimiser
  - Longitudinal integral of Gaussian at both ends
    - Fringe length given by σ=6cm
- Off-midplane fields extrapolated by repeated derivatives of this product of three functions

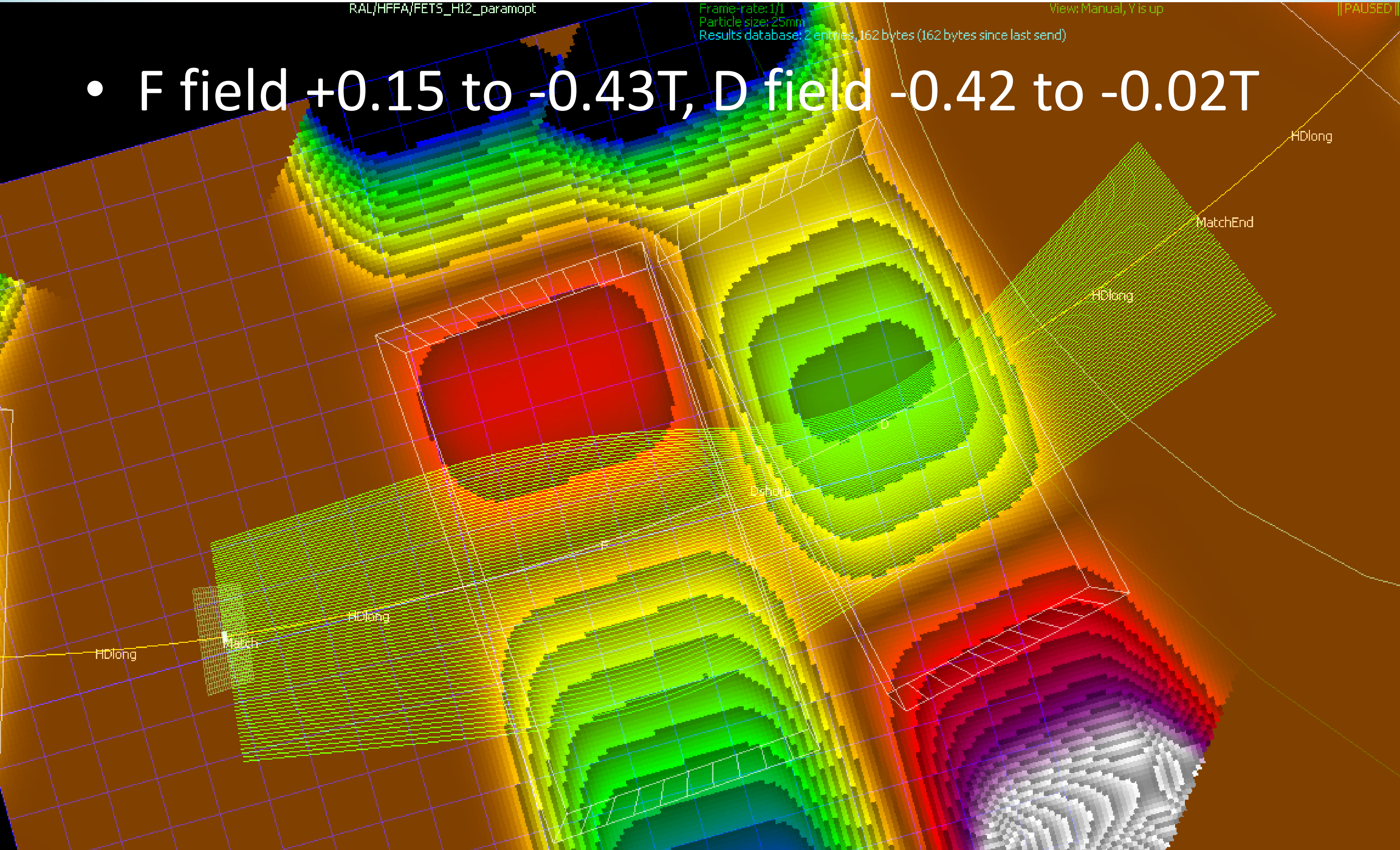
Fixed-Tune Cell Orbits

- Target cell tunes Q_x=0.216, Q_y=0.213



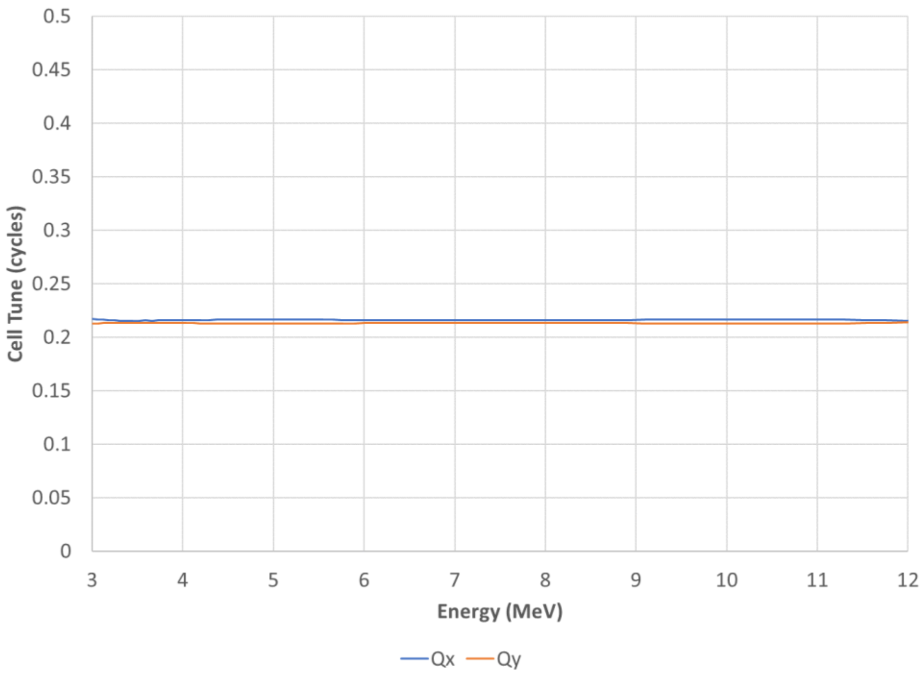
Fixed-Tune Cell Fields

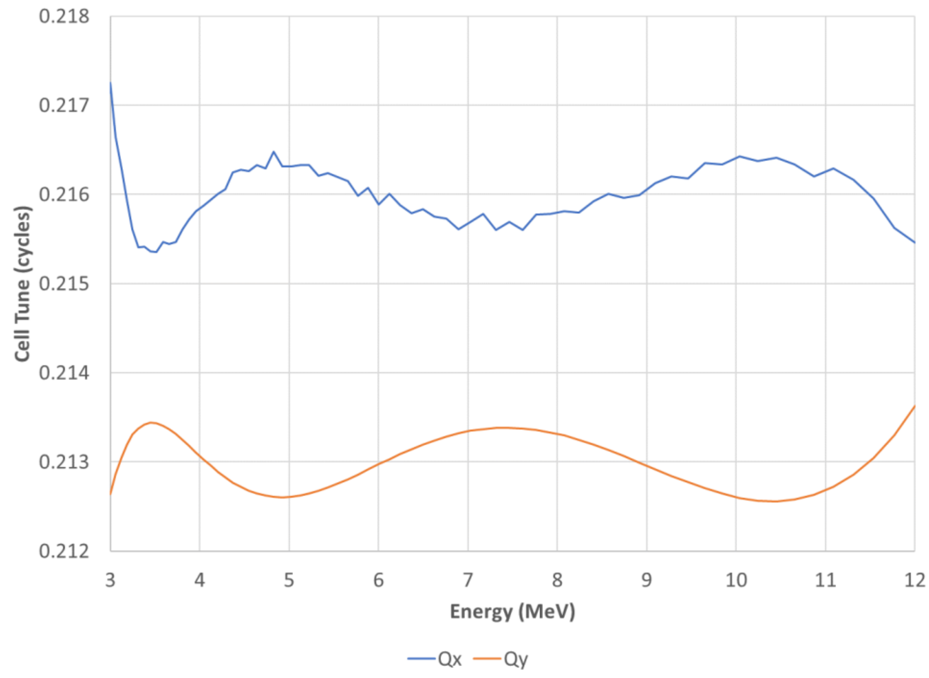
- F field +0.15 to -0.43T, D field -0.42 to -0.02T



Cell Tunes

- Target cell tunes Q_x=0.216, Q_y=0.213

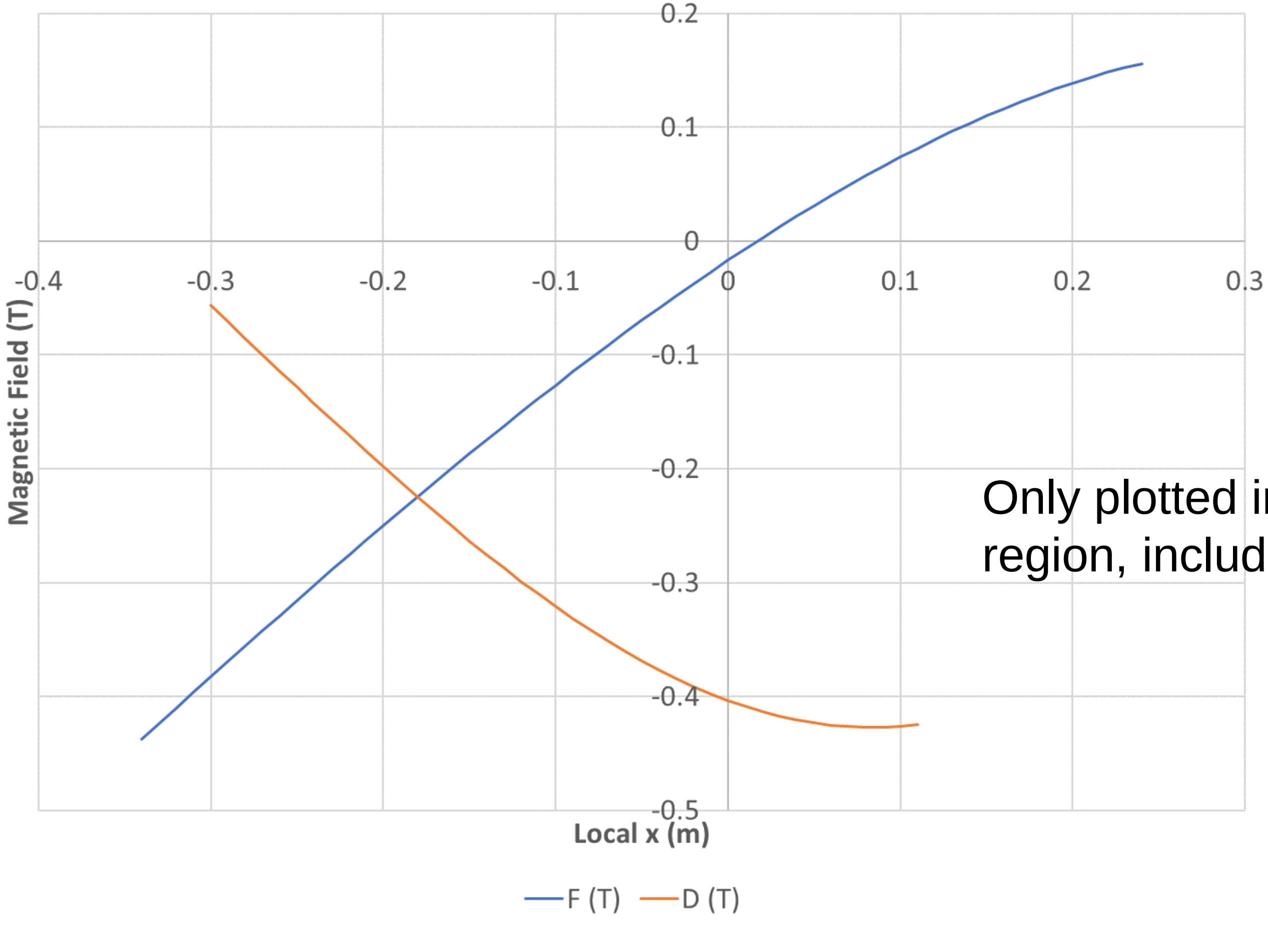




Field Profiles

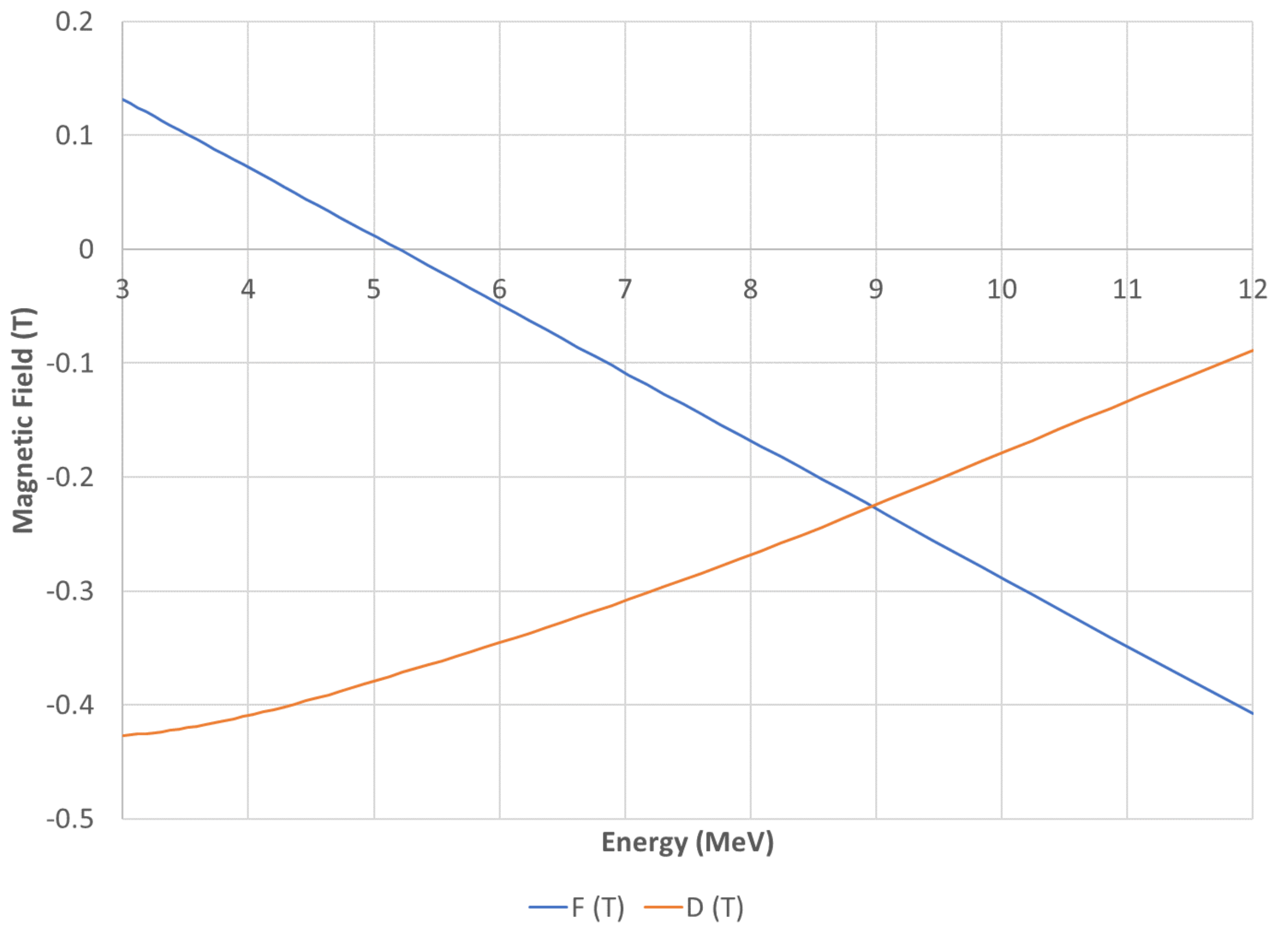
← 12MeV, ring outside

3MeV, ring inside →

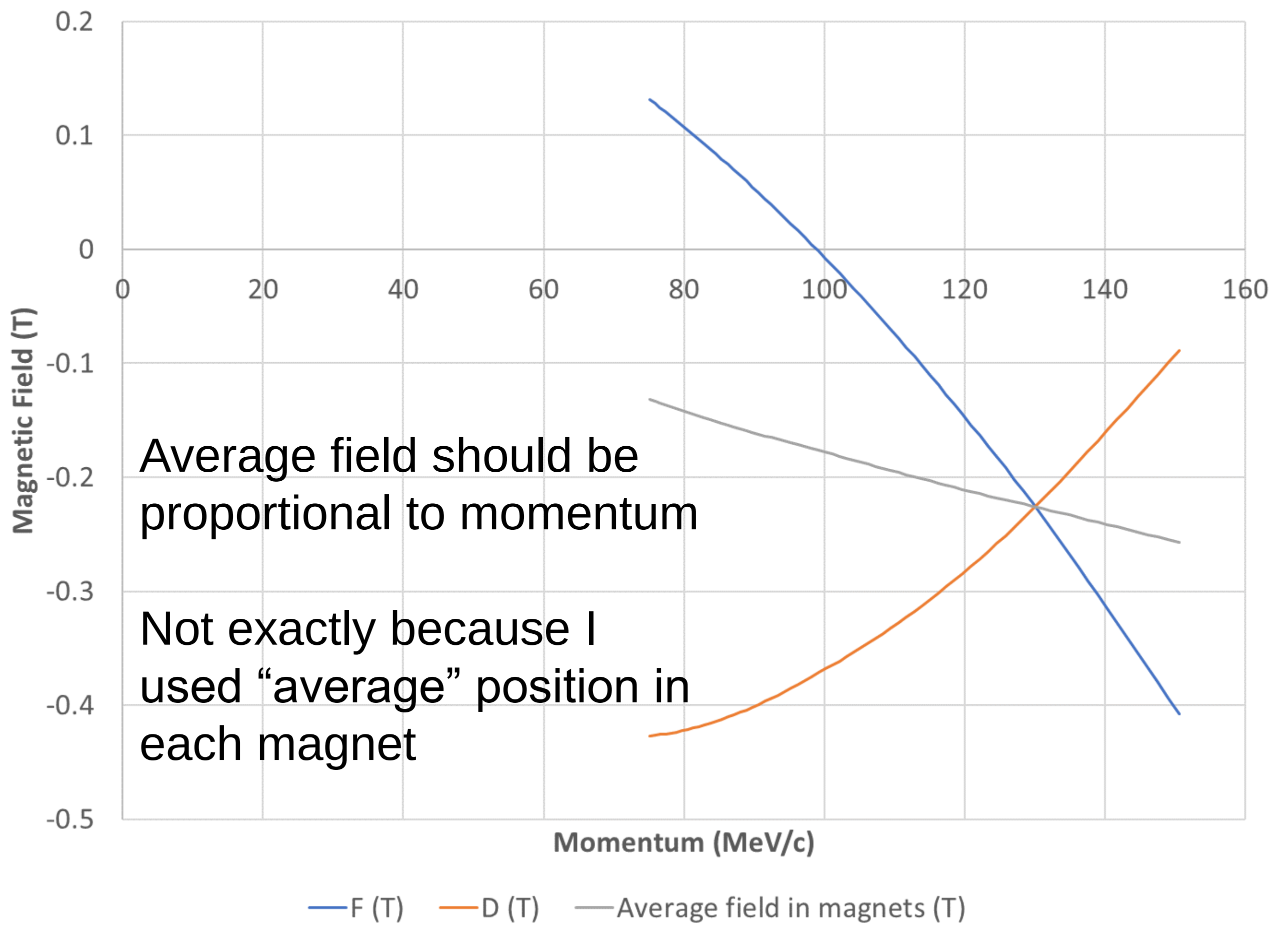


Only plotted in beam region, including sagitta

Field vs. Kinetic Energy



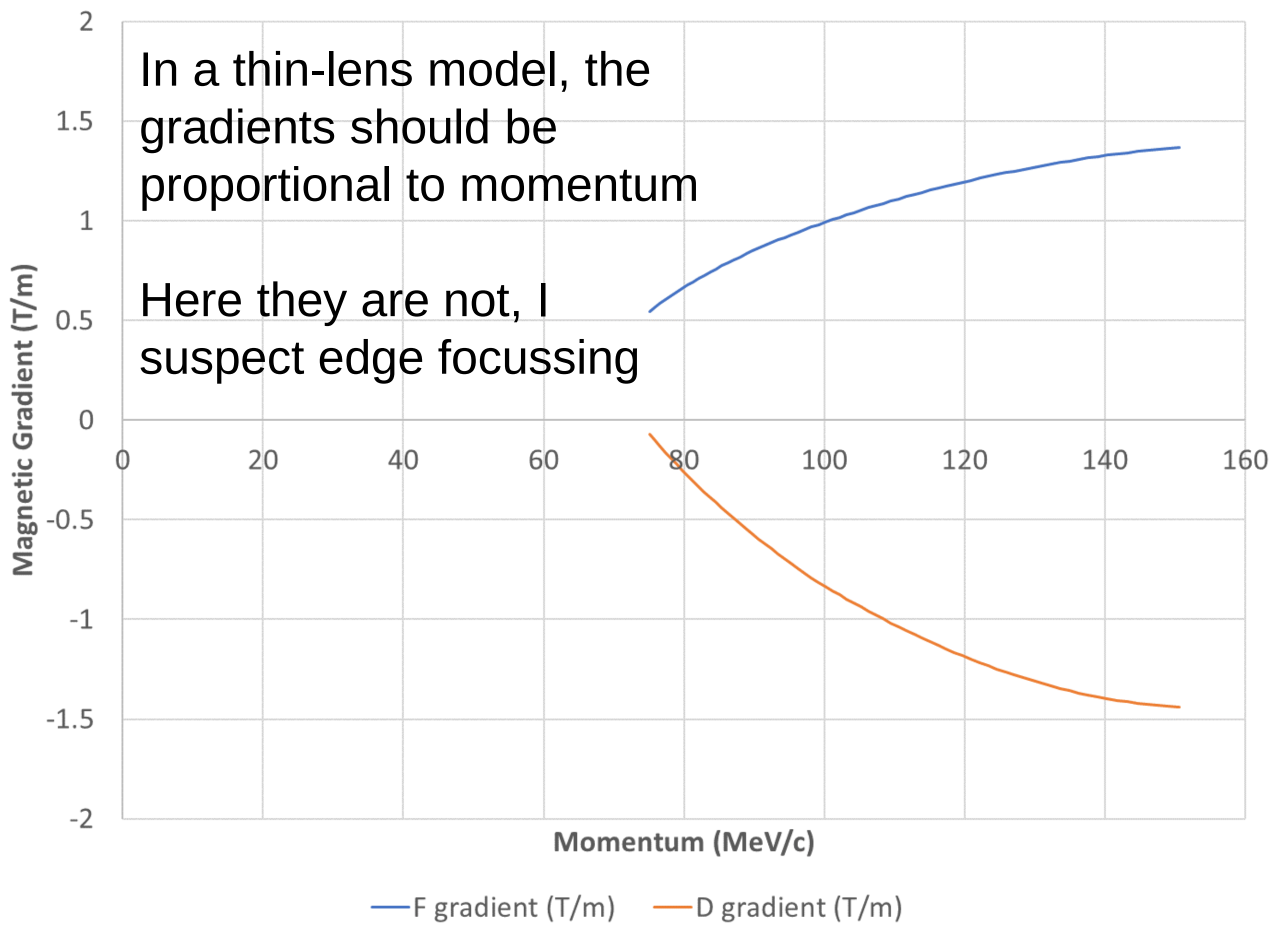
Field vs. Momentum



Average field should be proportional to momentum

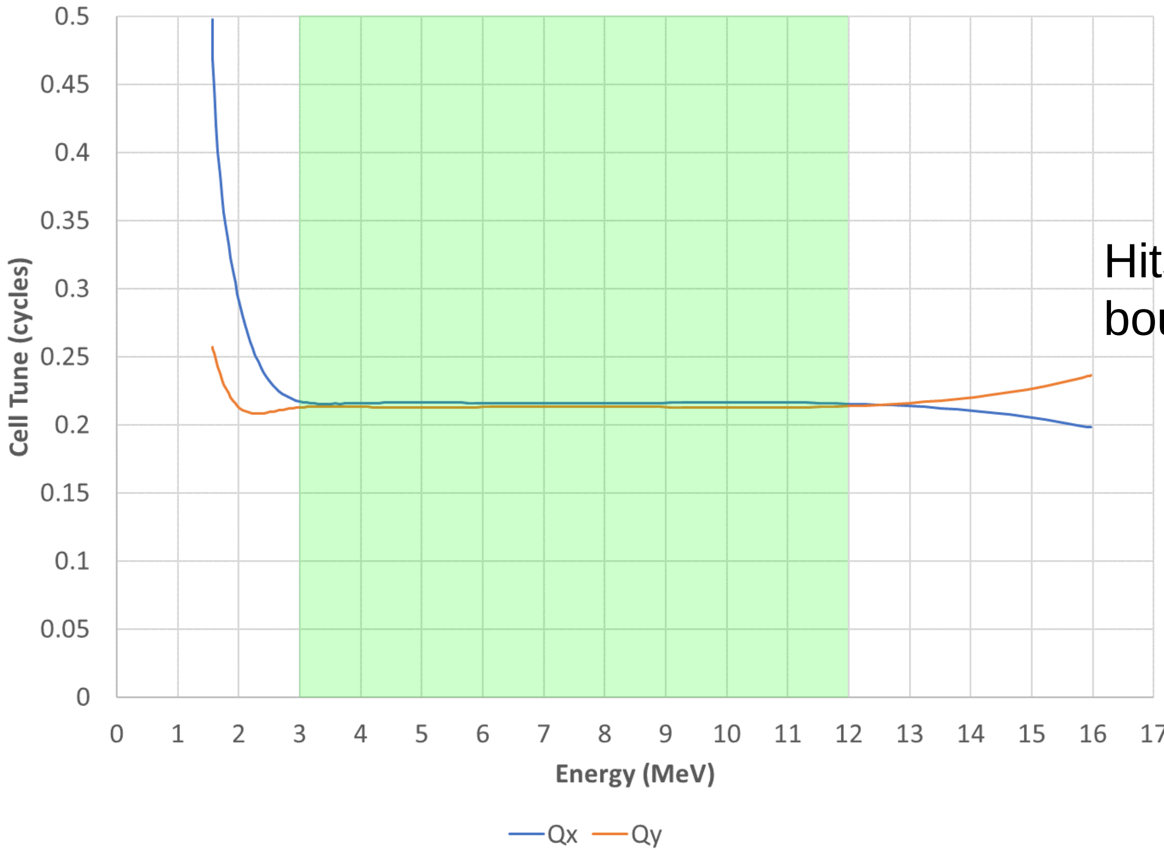
Not exactly because I used “average” position in each magnet

Gradient vs. Momentum



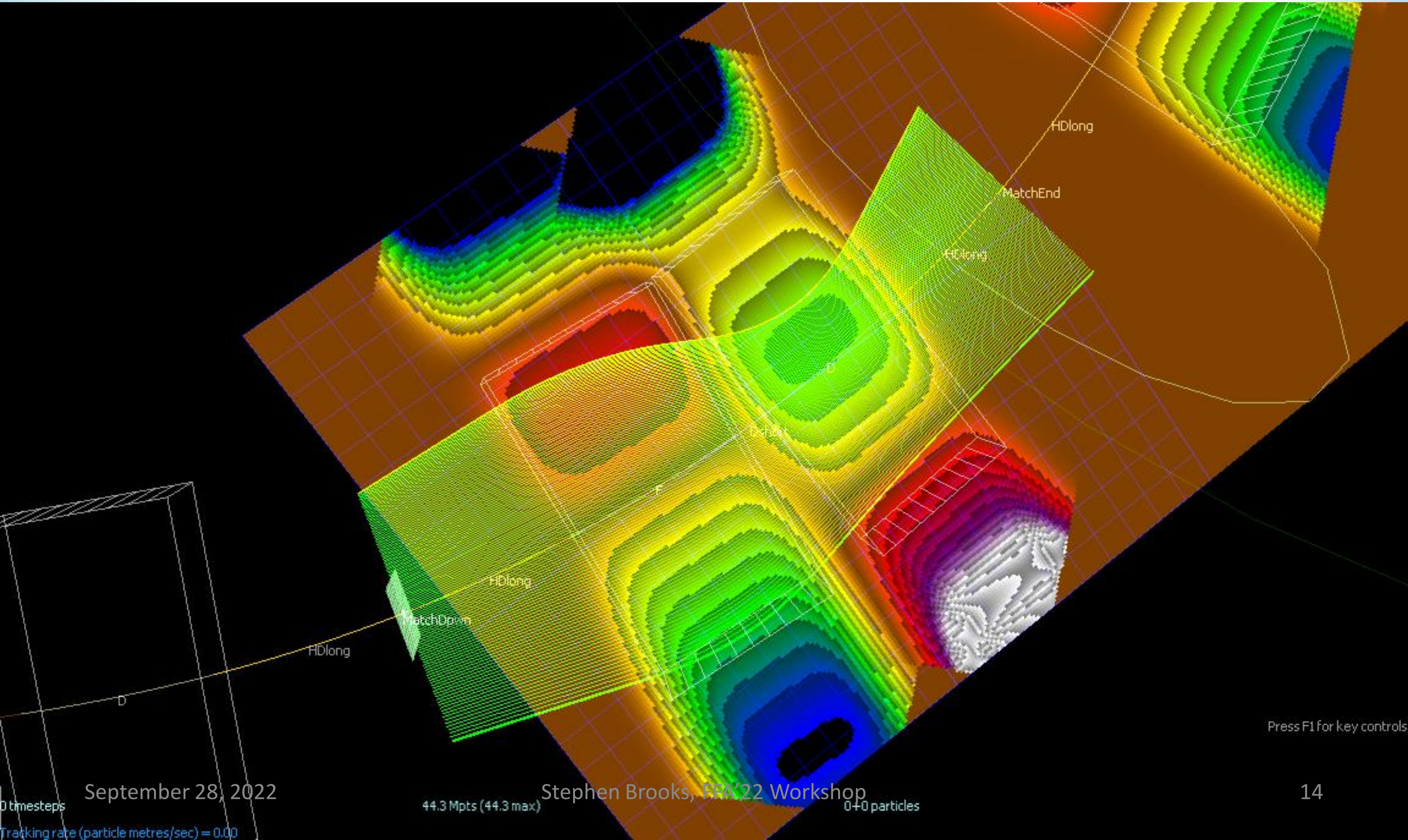
Tunes Beyond Energy Range

- Target cell tunes Q_x=0.216, Q_y=0.213

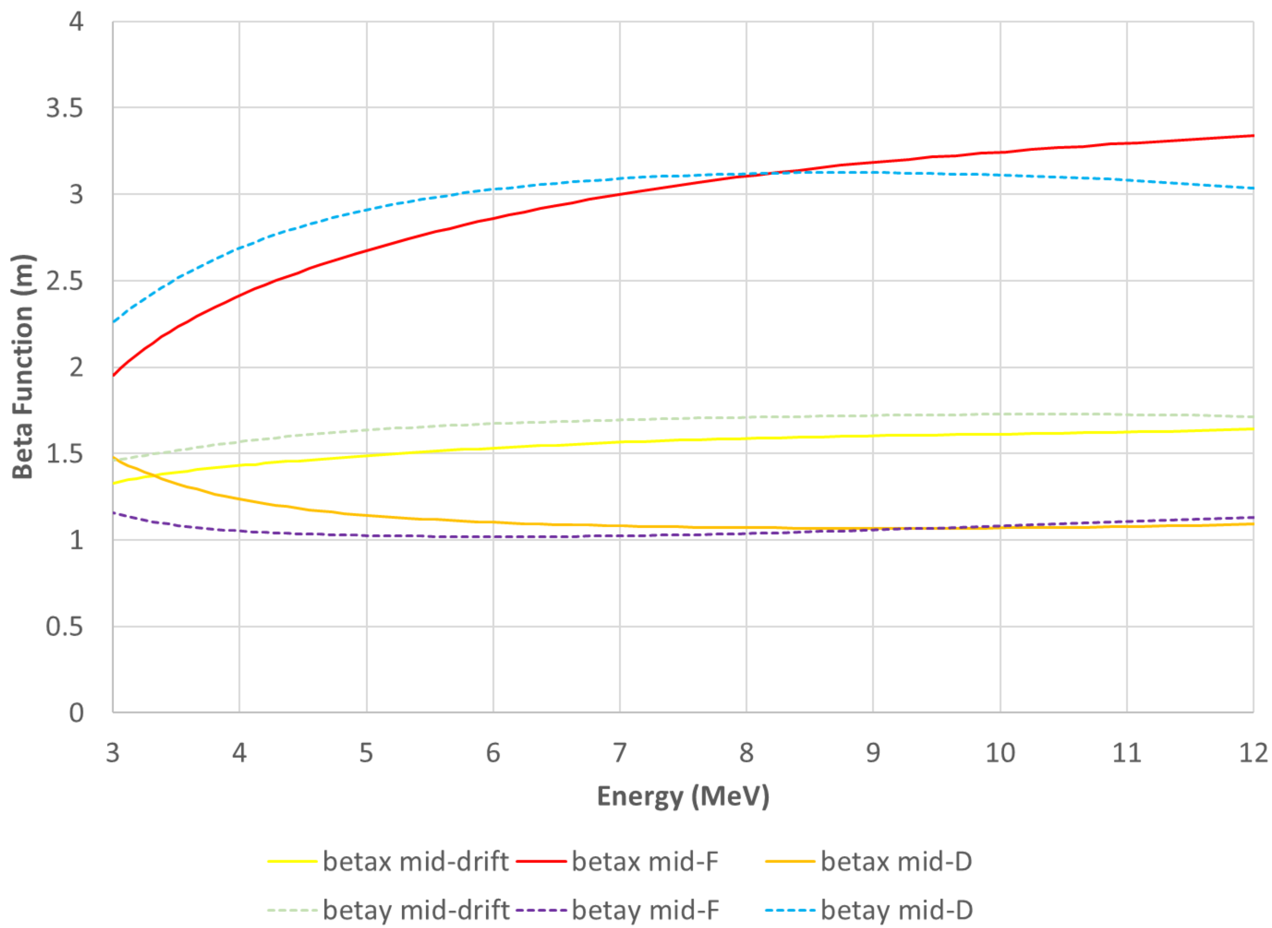


Hits magnet bounding box

Orbits Beyond Energy Range



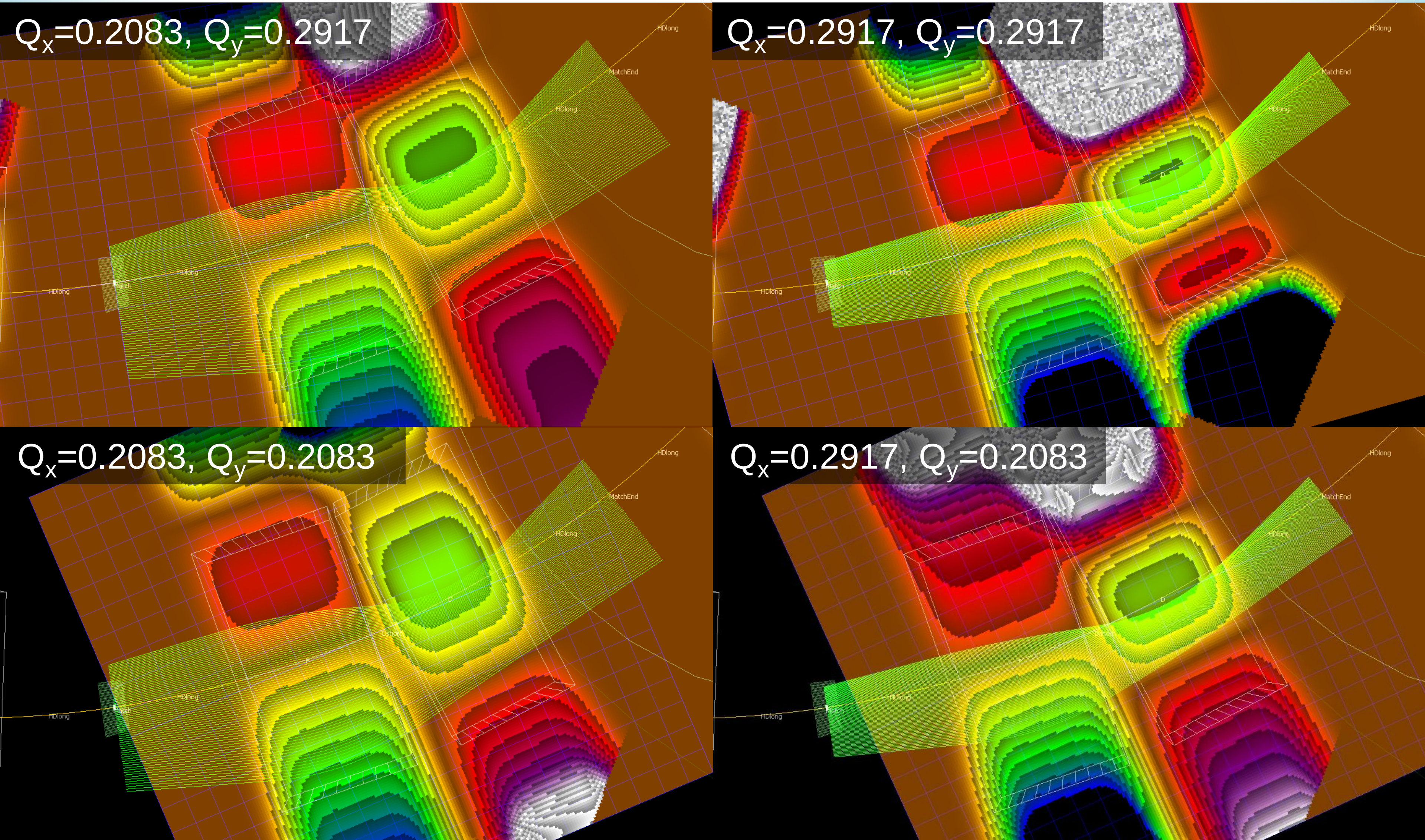
Beta x,y vs. Kinetic Energy



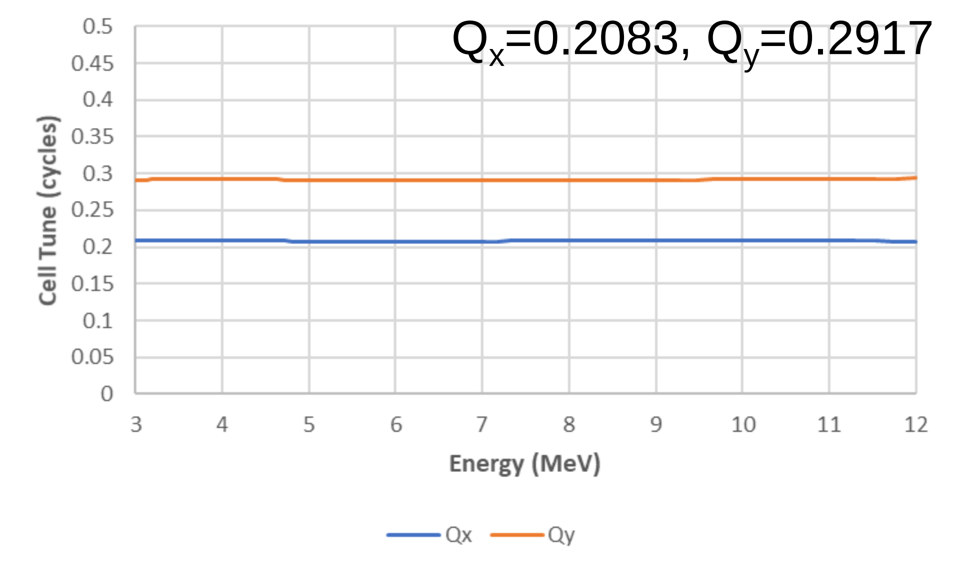
Tune Range Requirement

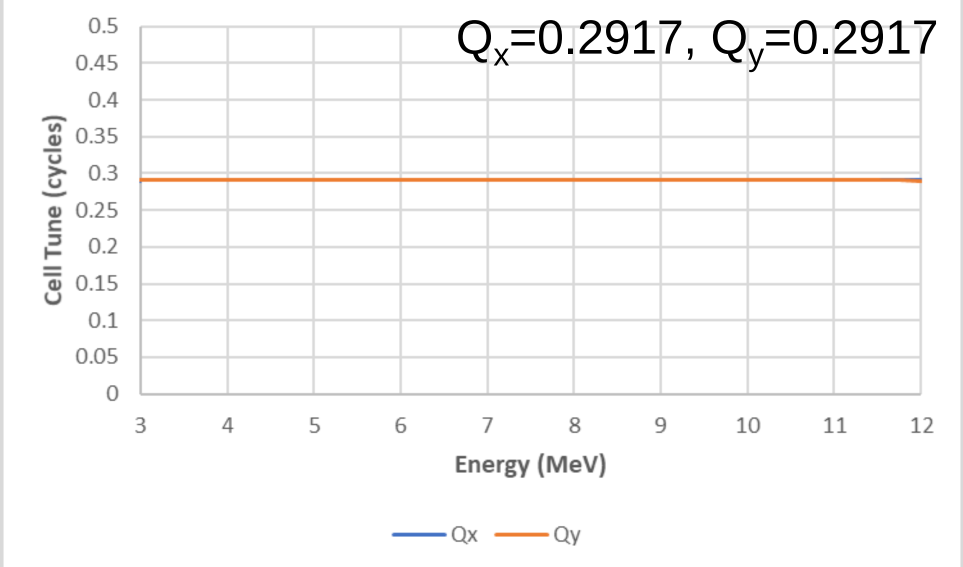
- $Q_{x,y}$ tunable over full range of 1 in ring
  - For exploration of the tune plane in R&D
  - I chose 12 cells so this is 1/12 range per cell
- Q_x=0.216, Q_y=0.213 would be ring tune of (2.592,2.556)
- So choose 2.5 to 3.5 ring tune range in both planes
  - 0.2083 to 0.2917 cell tune (all combinations)

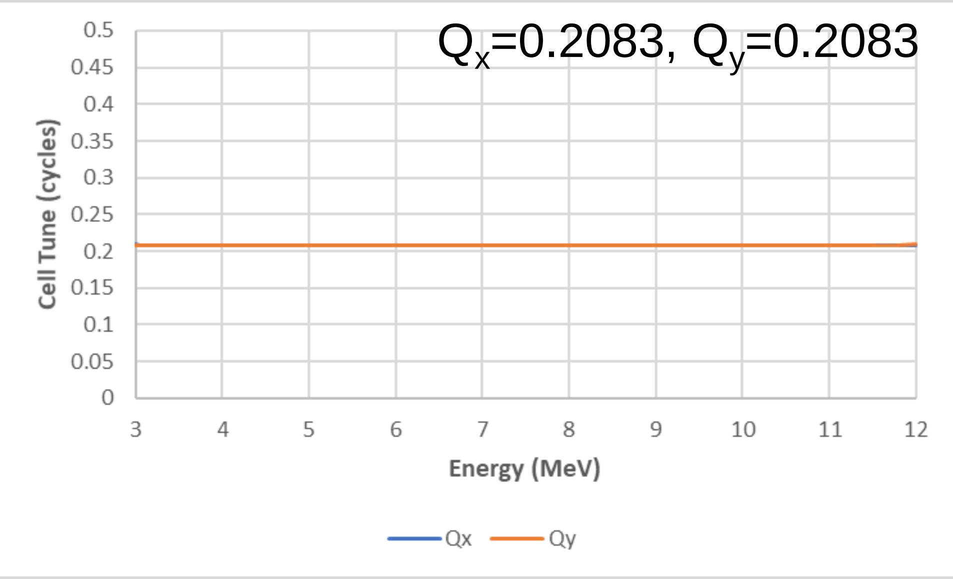
Four Tune "Corners"

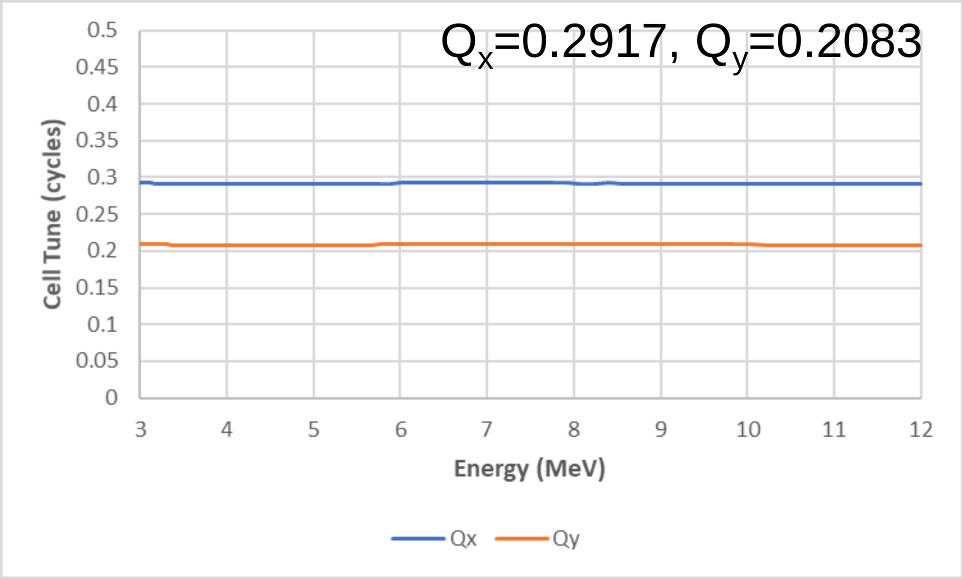


Cell Tunes at Corners

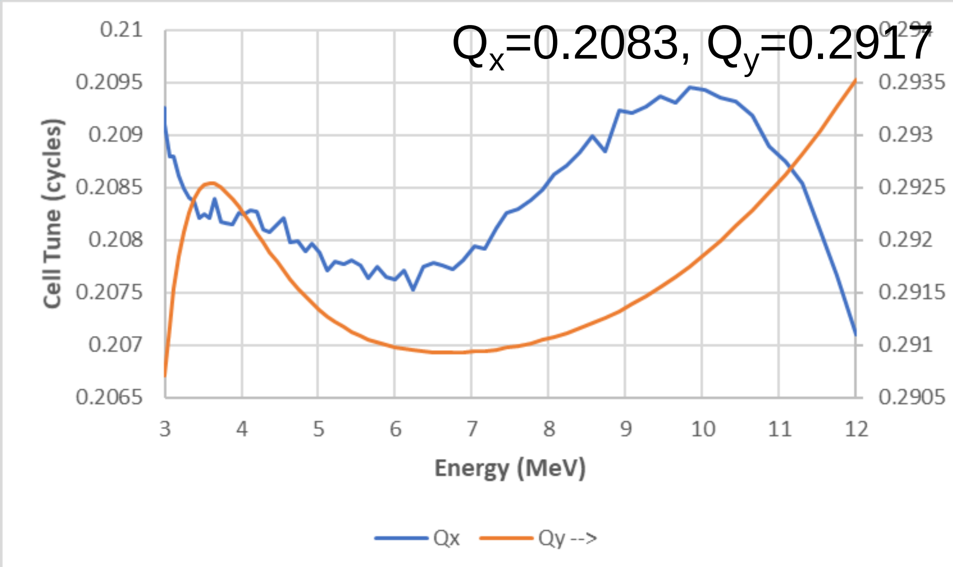


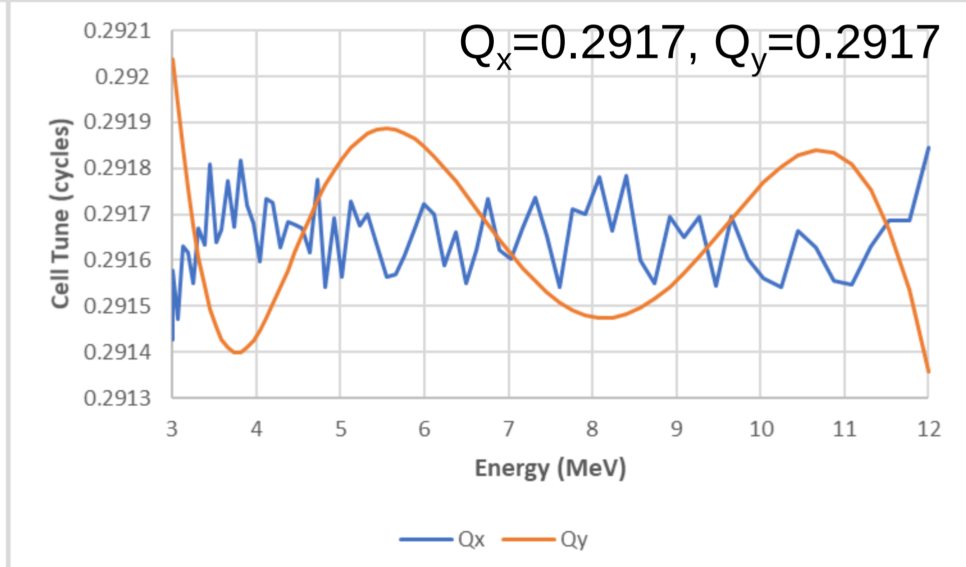


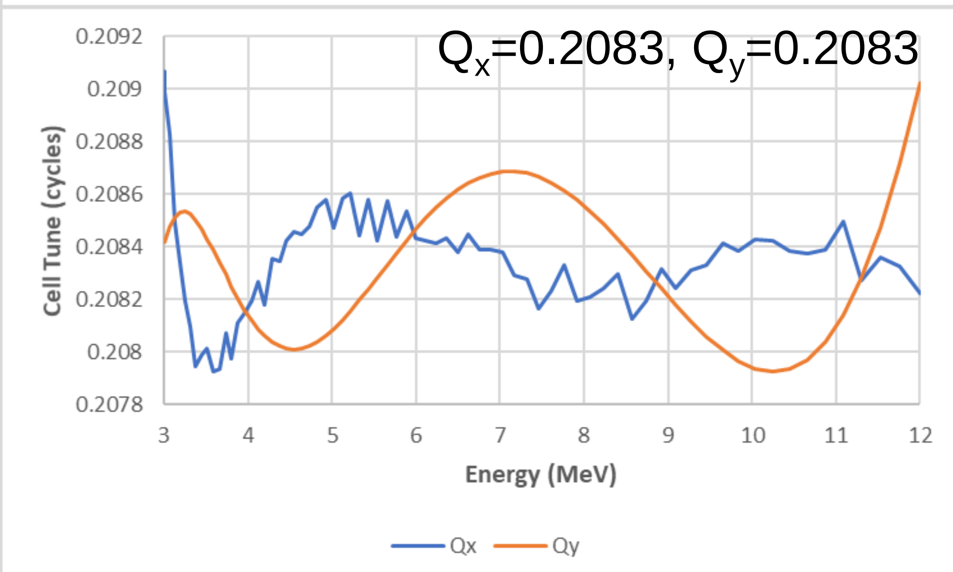


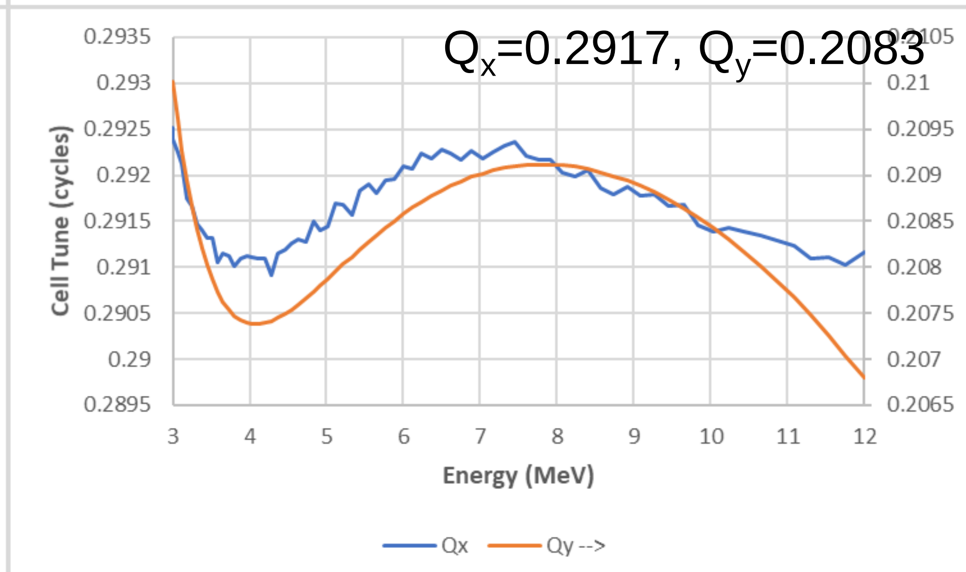


Cell Tunes at Corners (zoom)

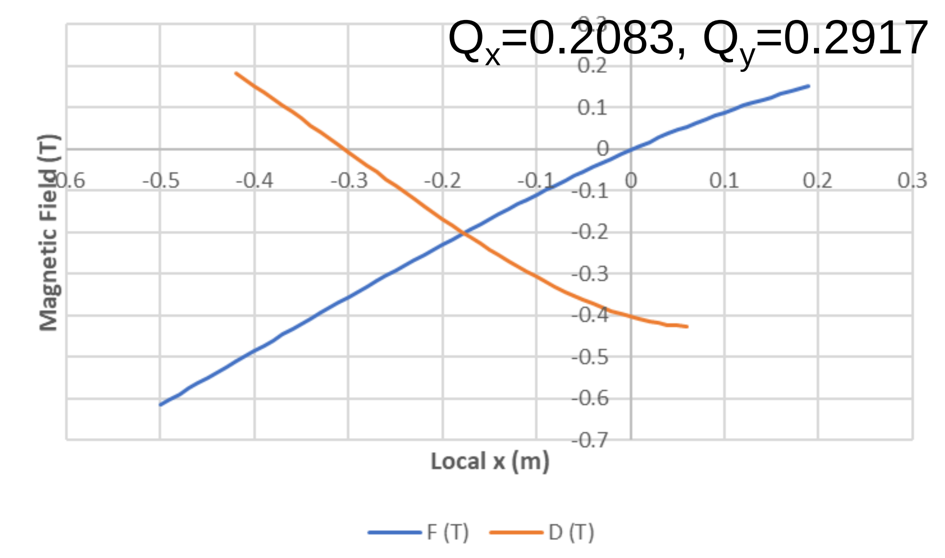


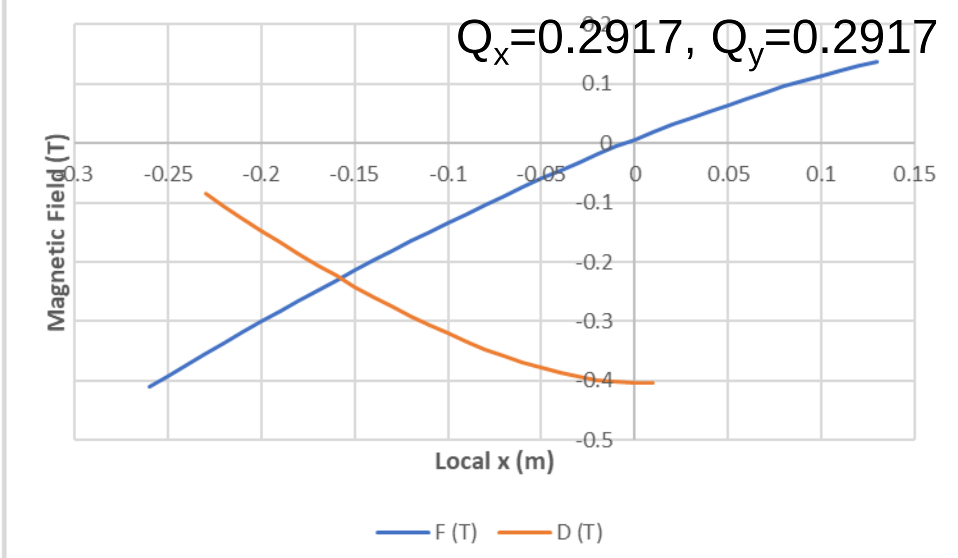


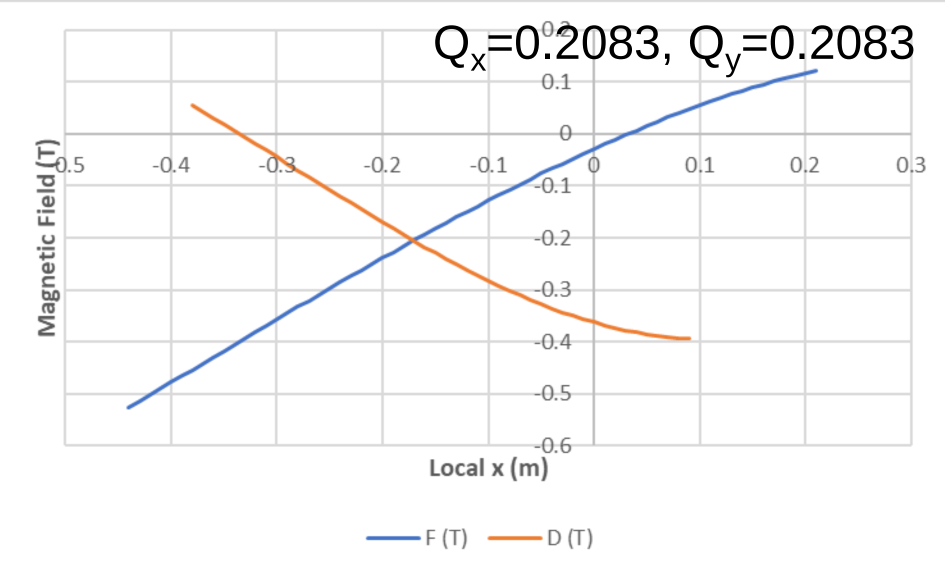


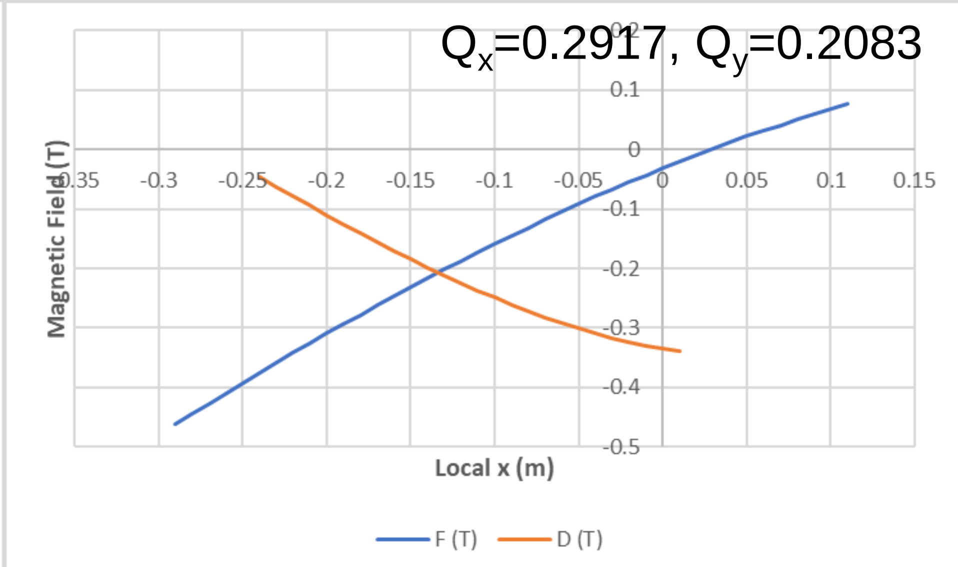


Field Profiles of Corner Designs









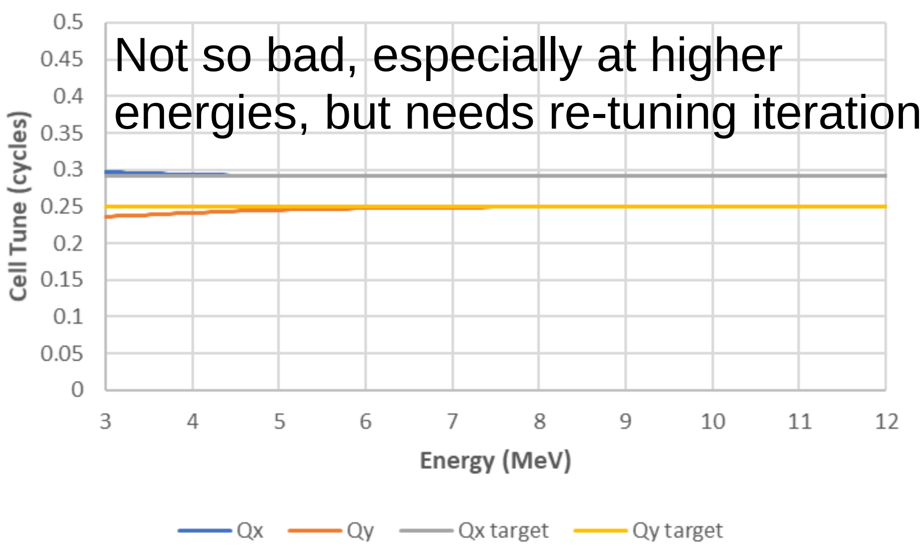
Designs Summary Table

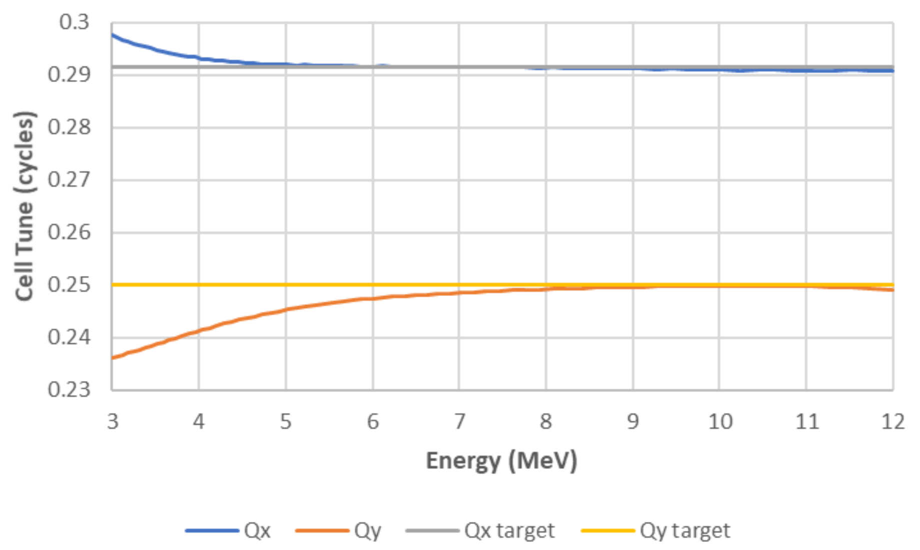
Cell Q_x	Cell Q_y	Max tune error	Max field (T)	Orbit range (m)
0.2083	0.2083	0.00074	0.5204	0.641
0.2083	0.2917	0.00185	0.6128	0.688
0.2917	0.2083	0.00168	0.4466	0.389
0.2917	0.2917	0.00037	0.4004	0.381
0.216	0.213	0.00126	0.4310	0.573

- Q_x low, Q_y high corner is most "unnatural"
  - Largest orbit excursion, highest field
    - Had to let optimiser vary dipole to stay in magnet box
- Higher Q_x tunes greatly improve orbit range

Half-way Interpolation Result

- What happens if you average the <u>fields</u> (equivalently, multipole coefficients) between two of the corner designs?
  - (0.2917,0.2083),(0.2917,0.2917) → (0.2917,0.25)?



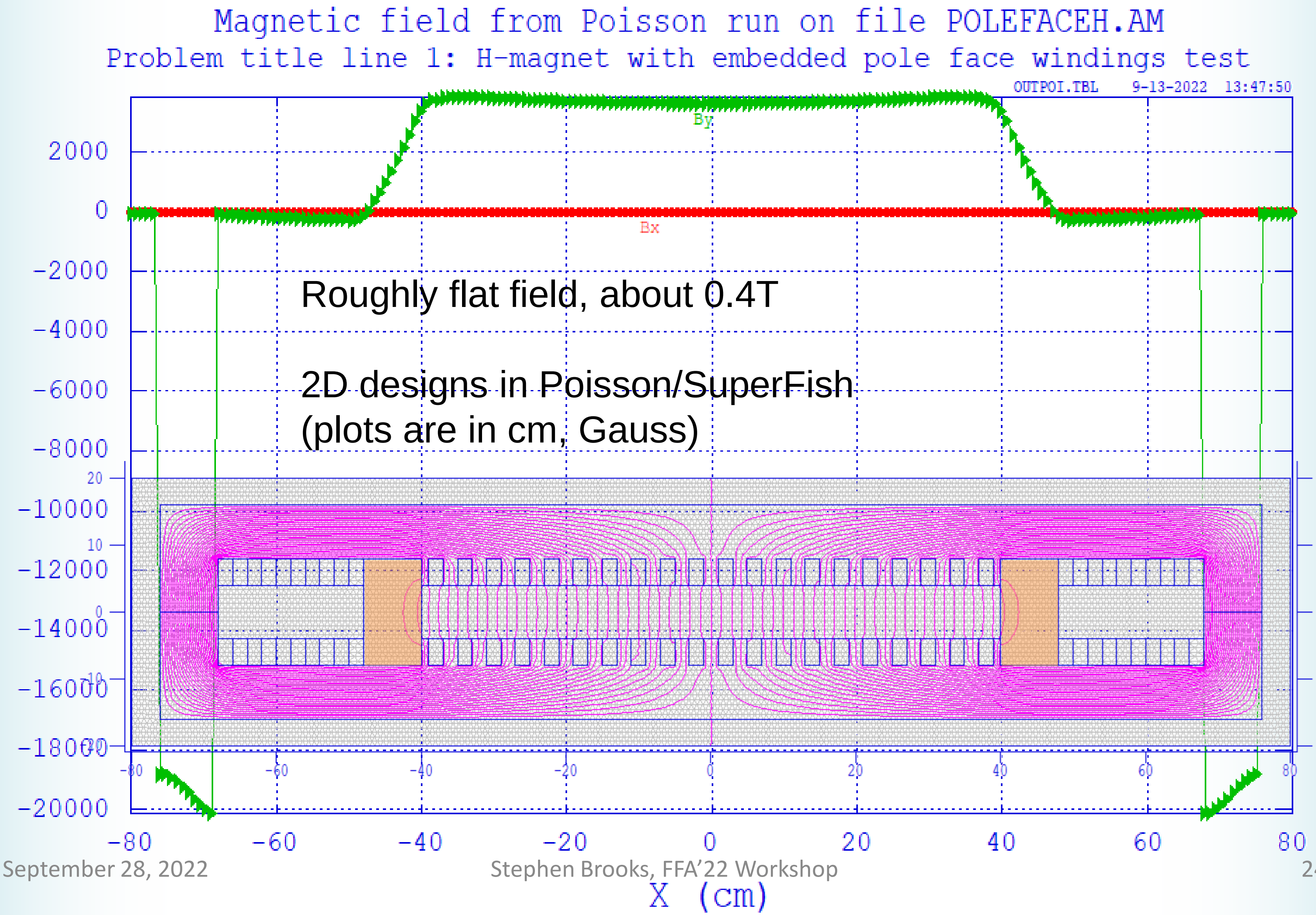


Horizontal Omni-Magnets

- "Arbitrarily" configurable field $B_y(x)$
  - Are their own correctors
- Try flat 80cm-wide poles, 8cm full gap, with large embedded pole face windings
  - 20 windings each top and bottom
  - 2x4cm cross-section, 4cm horizontal pitch
- Main 8x16cm H-magnet-style coil for dipole
- Pole face windings return on nearest side
  - Produce quadrupole if all have equal current

Round ones were described in my IPAC'13 paper

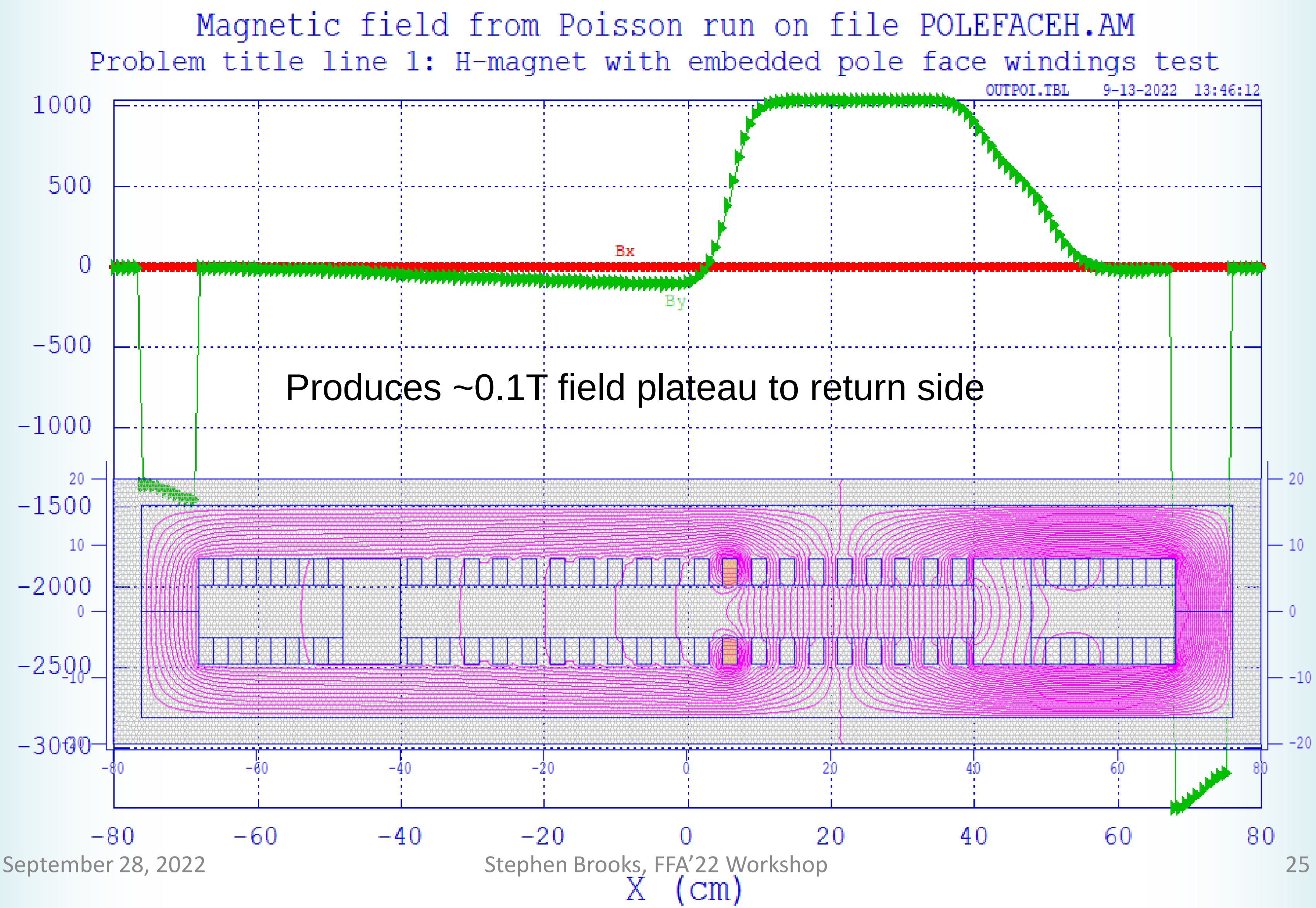
Main Dipole Coil @ 5A/mm^2



Roughly flat field, about 0.4T

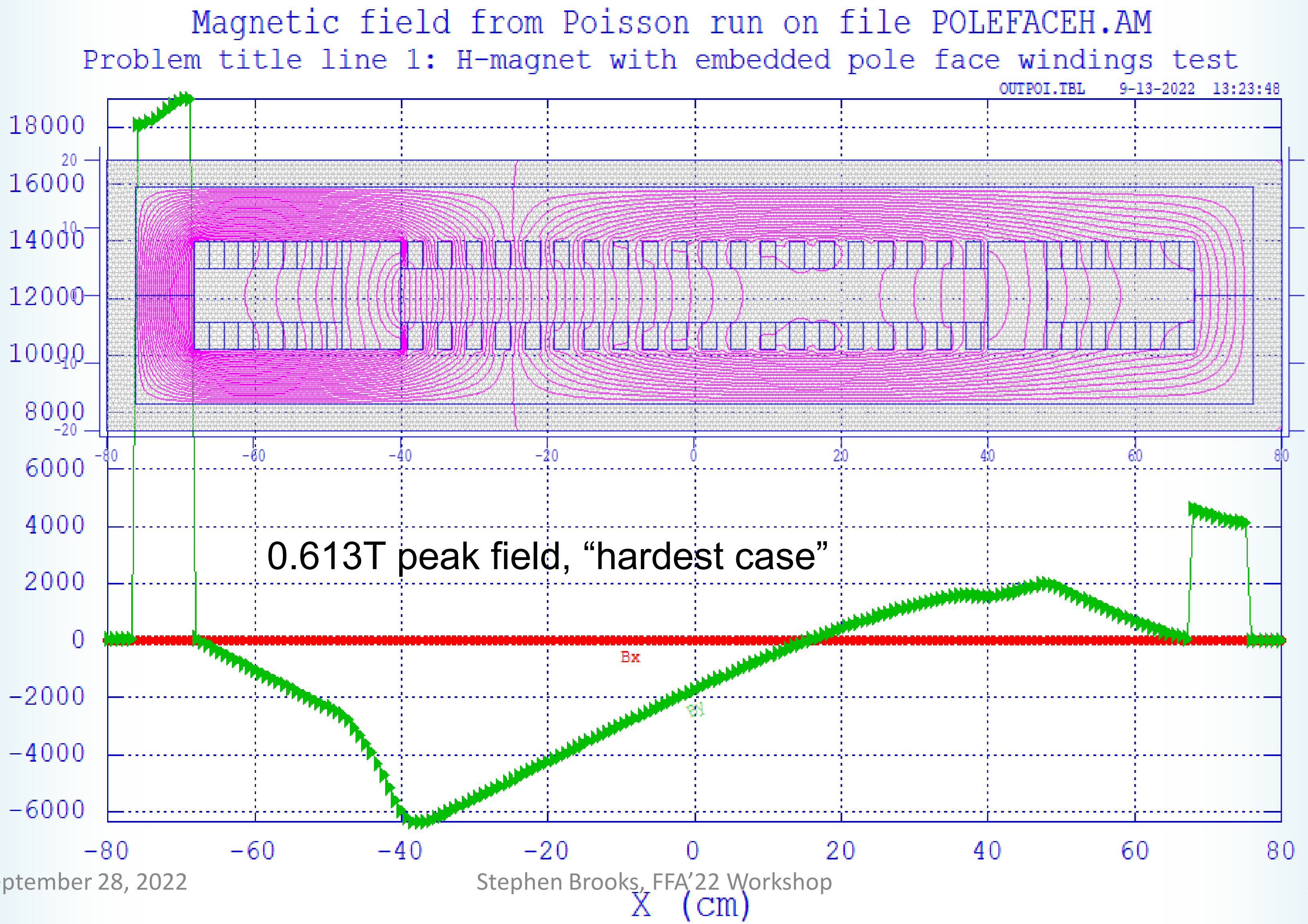
2D designs in Poisson/SuperFish (plots are in cm, Gauss)

Single Pole Winding @ 5A/mm^2



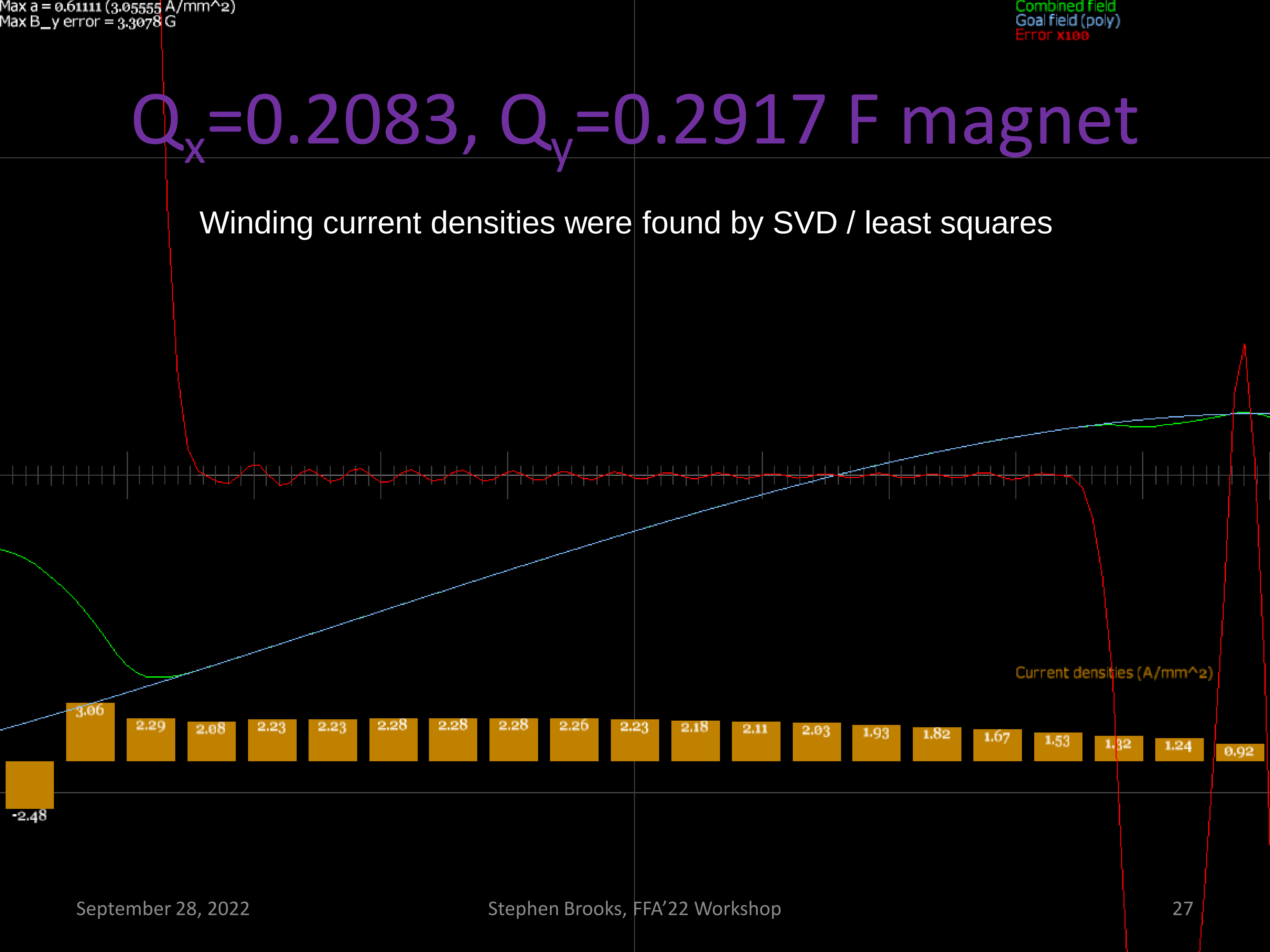
Produces ~0.1T field plateau to return side

Q_x=0.2083, Q_y=0.2917 F magnet

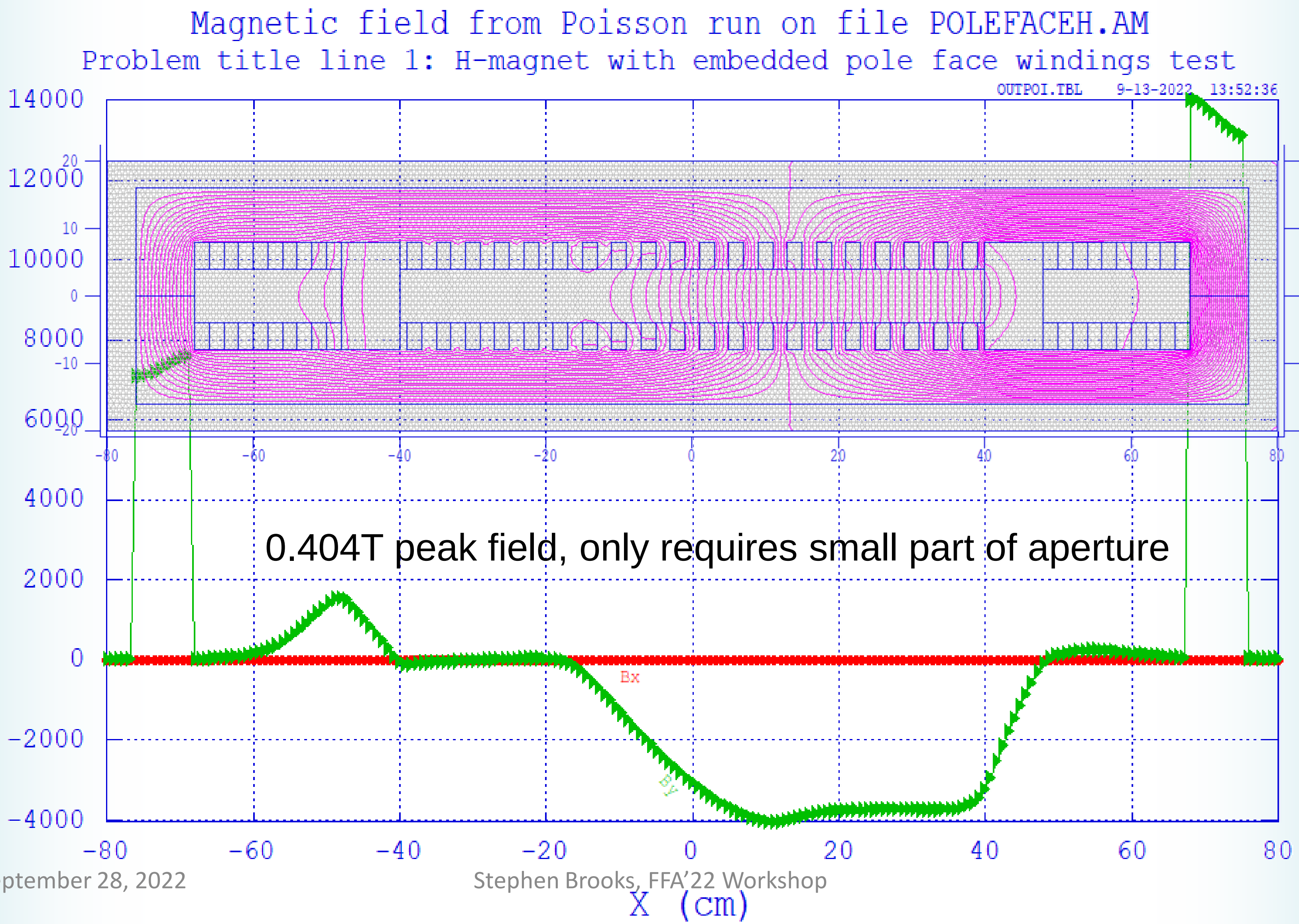


Q_x=0.2083, Q_y=0.2917 F magnet

Winding current densities were found by SVD / least squares



Q_x=0.2917, Q_y=0.2917 D magnet



Q_x=0.2917, Q_y=0.2917 D magnet

SVD only activates coils in the region they have an effect

Magnets Summary Table

Cell Q_x	Cell Q_y	Magnet name	Max current density (A/mm²)	Max B_y error (G)	Max field (T)	Orbit range (m)
0.2083	0.2083	F	2.090	2.25	0.5201	0.641
		D	3.111	1.98	0.3921	0.460
0.2083	0.2917	F	3.056	3.31	0.6126	0.688
		D	2.955	2.31	0.4260	0.471
0.2917	0.2083	F	3.073	2.23	0.4418	0.389
		D	3.189	1.89	0.3370	0.247
0.2917	0.2917	F	3.480	2.50	0.3883	0.381
		D	3.944	2.58	0.4035	0.224
0.216	0.213	F	2.372	1.88	0.4224	0.573
		D	4.256	2.32	0.4270	0.400

Quick Dynamic Aperture Test

- Started 8 particles at (x,y) = (±2.5/0,±2.5/0)cm
  - Cell Q_x=0.216, Q_y=0.213 design
- Tracked in ring for 60μs, all survived
- Some particles at 3cm did not
- Can use $\varepsilon = x_{inj}^2/\beta_{inj}$ to get apertures

Energy	Cells (in 60μs)	Turns	ε_x (geom.)	ε_y (geom.)
3MeV	697	58	471	428 mm.mrad
6MeV	954	79	408	374 mm.mrad
12MeV	1266	105	381	365 mm.mrad

Dynamic Aperture Notes

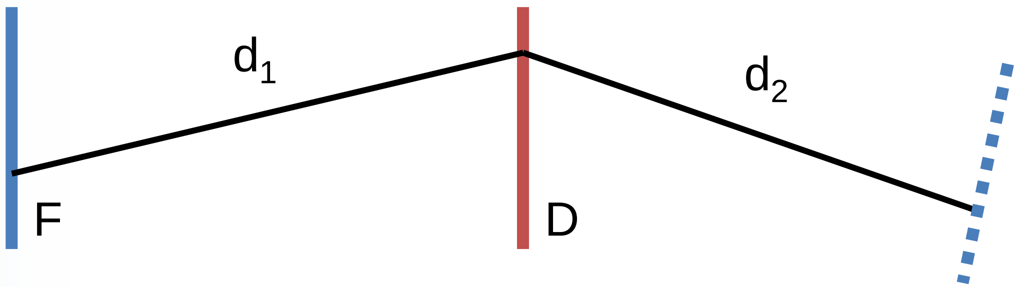
- FETS-FFA request was 100mm.mrad normalised at 3MeV ($\beta\gamma$=0.080)
  - = 1250 mm.mrad geometric
- β_y = 1.46m at 3MeV injection point
  - Implies y = ±4.27cm (or ±5.31cm at maximum β_y)
  - Larger than the magnet aperture (±4cm)!
- This DA is not the painted beam size, it is a stability margin (all beam ≤ 250 mm.mrad)

Analytic Method for 2 Thin Lenses

- Assume small-angle (paraxial) approximation
  - Weak focussing and edge focussing are negligible
  - Not valid for the FETS-FFA example!
- Fixed cell tunes with two lenses implies constant normalised focussing strength with momentum in each lens
  - Q_x, Q_y are two constraints on linear optics and the focussing strengths (gradients) are two variables

For the case with 3 lenses and 2 constraints, see my IPAC'11 paper

Function Variable Elimination



Ends at θ_{cell}, Δx_{cell} (constants)

$x_F(p)$ $x'_1(p)$ $x_D(p)$ $x'_2(p)$

$\theta_F(p)$ $\theta_D(p)$

$b_F(x)$ $b_D(x)$

Four constraints from linear tracking at each energy (and closure)

Two constraints from constant normalised gradient assumption

$$b'_e(x_e(p)) = k_e p$$

...becomes the differential equation

Two constraints from rigidity relation

Normalised b = q(Bℓ)

$$\theta_e(p) = \frac{b_e(x_e(p))}{p}$$

Change to Geometric Variables

- Rigidity: $p\theta(p) = b(x(p))$
- Constant norm. gradient: $b'(x(p)) = kp$
- dRigidity/dp: $\theta(p) + p\theta'(p) = b'(x(p))x'(p)$
- Therefore: $\theta(p) + p\theta'(p) = kpx'(p)$ [$= kD_x(p)$]
- x dependency and b have been eliminated
- The rest is easy
  - Linear algebra
  - Solving a 1-variable differential equation

See https://stephenbrooks.org/ap/report/2022-3/fixedtune2.pdf

Two Thin Lenses Result

- The general solution:

$$A = \frac{k_D + k_F}{-k_D - k_F - k_F k_D \frac{d_1 d_2}{d_1+d_2}}$$

$$B = \frac{-k_F \theta_{\text{cell}}}{-k_D - k_F - k_F k_D \frac{d_1 d_2}{d_1+d_2}}$$

$$C = \frac{\theta_{F0} + \frac{B}{A}}{p_0^A}$$

$$\theta_F(p) = C p^A - \frac{B}{A}$$

$$x_F(p) = \frac{(1+A)C}{A k_F} p^A - \frac{B}{A k_F} \ln p +$$ (arbitrary choice of x origin)

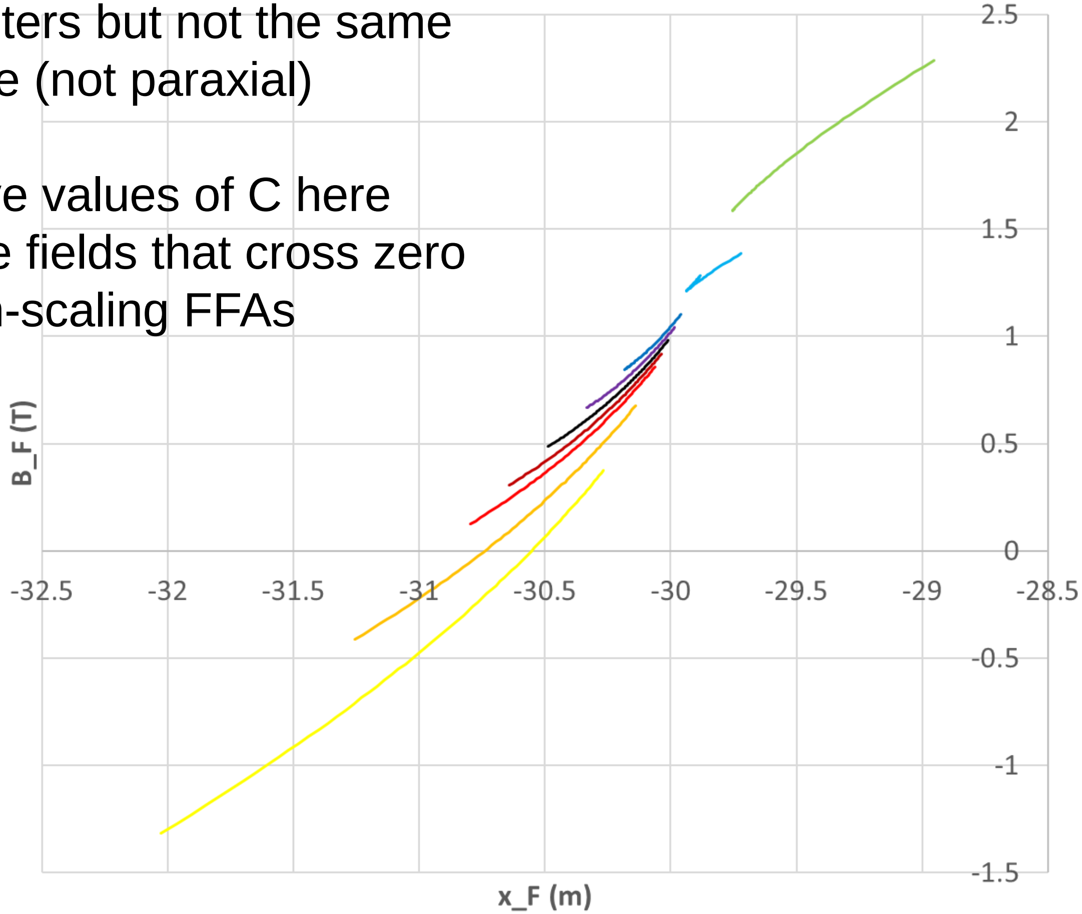
- C=0 corresponds to straight scaling FFA
  - Constant angle, logarithmic orbit position
- Range of other solution families are possible
- $b_F(x_F(p)) = p\theta_F(p)$ gives field profile implicitly

Example

d1	0.597198	m
d2	1.497198	m
thcell	0.523599	rad
kF	1.497447	m^-1
kD	-0.73041	m^-1
dd/d+d	0.426912	m
A	-2.55594	
B	2.612672	

Approximately the FETS-FFA parameters but not the same machine (not paraxial)

Negative values of C here produce fields that cross zero like non-scaling FFAs



C values:
- 0
- -1.00E-50
- -2.00E-50
- -5.00E-50
- -1.00E-49
- 1.00E-50
- 2.00E-50
- 5.00E-50
- 1.00E-49

Conclusion

- Found fixed tune lattices to ~0.001 in cell tune
  - And automated: eliminated most hand-tuning!
- Dynamic aperture low by factor 2-3× in ε
  - Not small by most standards: >400mm.mrad
  - Haven't explored the full tune plane yet
- Magnetic efficiency about twice as good as (spiral) scaling FFA, magnets buildable
  - ~0.6T max field on beam vs. ~1.2T
- **Very interesting** machine type

Future Work

- Dynamic aperture search of tune plane
  - And maybe other parameters like fringe length or magnet edge angles (rect vs. sector etc.)
- Try expressing transverse field as something better-conditioned than polynomials
  - Like Fourier series, or Chebyshev polynomials
- Maybe investigate low-level tune “noise” in Muon1 tracking/optics algorithm
  - Could also be field model nonsmoothness